AN INVESTIGATION OF MACHINE REQUIREMENTS
FOR A MANUFACTURING FACILITY

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FOREWORD

Interest in this research resulted from experience in the area of facility planning and design and dissatisfaction with the available methods for estimating machine and equipment requirements. Dr. Rocker T. Staton and Dr. Harrison M. Wadsworth provided encouragement in investigating means of further quantifying machine estimating procedures.

Dr. Joseph J. Moder, Dr. John D. Neff and Dr. Wadsworth were helpful in review of, and suggestions for, the formulation of the statistical aspects of the research. Discussions with Dr. Staton, Professor Edward C. Franklin and Professor Winfield A. Brooks were of assistance during the period in which the scope of the research was being defined.

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Finally, to my wife Ginny, daughter Jackie, and son Ricky, for their sacrifice in order that I could pursue a doctoral program and their encouragement during periods of depression, I express special thanks.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

SUMMARY

The purpose of this research has been to develop a procedure for determining machine or equipment requirements for a manufacturing facility. Although applicable for the equipment to be used for any production system, it is designed primarily to satisfy requirements in job-shop or intermittent type manufacturing. For continuous type manufacturing, available line balancing techniques are more efficient.

Previously available models for determining machine requirements have been limited to conditions of certainty for the input variables. Morris derived a decision rule based upon optimized cost but did not define the distribution of machine requirements. The distribution is necessary for input to the decision model; here it is assumed to be approximately normal. This assumption is justified on the basis of several applications of the central limit theorem. Expressions for the parameters of the distribution are then derived.

There is wide variation in methods of forecasting sales requirements by industry. This factor was therefore not considered in the model developed in this thesis. Sales requirements were taken to be fixed and known.

In addition to the recognition of machine requirements as a random variable, some traditional input variables are redefined. Average time rather than standard time is used to measure the operation time. The planning time interval is one year. The reciprocal of the upper bound of the equipment utilization factor is used to allow for the non-

operating time of the equipment. Non-operating time at this upper bound is divided into its major components; each component is then defined respectively as a constant or as a random variable. These components are then combined to provide an expression for the reciprocal of the upper bound.

By combining the distributions of the input variables, or their equivalents, the joint distribution of machine requirements is found to be approximated by a normal distribution with

$$E(N_T) = \frac{1}{H_a}\{\sum_{j=1}^{n} [\frac{E(N_j)}{Q_j} E(P_j \bar{t}_{s_j})] + \sum_{i+1}^{v} [\frac{H_a}{\Delta H_{t_{m_i}}} E(\bar{t}_{m_i})]$$

$$+ [E(m + 1) \sum_{j=1}^{n} (E(\bar{t}_{a_j} P_j))]\}$$

and

$$Var(N_t) = (\frac{1}{H_a})^2 \{E(m + 1)^2 Var \sum_{j=1}^{n} (\bar{t}_{a_j} P_j) + E (\sum_{j=1}^{n} (\bar{t}_{a_j} P_j))^2$$

$$Var(m) + \sum_{j+1}^{n} \frac{E(N_j)^2}{Q_j} Var (P_j \bar{t}_j) + \sum_{i=1}^{v} \frac{H_a}{\Delta H_{t_{m_i}}}^2 Var(\bar{t}_{m_i})\}$$

where

$E(N_T)$ = The expected value of the number of machines for a single model.

H_a = Actual hours per time period per machine.

$\bar{t}_{s_j}$ = Average set-up time required for operation j.

$E(N_j)$ = Expected value of number of machines for operation j.

Q_j = Size of production lot at operation j.

P_j = Required number of cycles per time period for operation j.

$\bar{t}_{a_j}$ = Average time for operation j.

v = The total number of time interval scheduled preventive maintenance operations on the machine per unit time period.

$H_{t_{m_i}}$ = Hours between occurrences of maintenance operation i, scheduled on the basis of a fixed time interval.

m = The slope of the regression equation of equipment maintenance hours on equipment use hours per time period.

An illustration is used to compare the effect of the proposed model to the Shubin-Madeheim formulation, which assumes conditions of certainty. In this example, the mean use factor is approximately equal to the most frequently used value of the utilization factor for input into the Shubin-Madeheim formula, 0.85. This provides better comparison. The result is a mean value which is not significantly different from the result obtained using the Shubin-Madeheim formula. This similarity indicates the expected values by the proposed model, and the results using average values under the assumption of certainty are identical for equal mean values of the use factor. This similarity was expected due to the use of the Shubin-Madeheim equation as the initial model in the derivation.

The determination of the variance permits establishment of decision methods under criteria of minimized total cost and of machine availability. Economic decisions such as the cost minimization models of Morris may be used in conjunction with the derived distribution of machine requirements. Probabilities of machine availability may be computed from the derived distribution.

The proposed model provides a means for incorporating factors of uncertainty into the proper selection of the number of machines needed when designing a new, or revising an existing, manufacturing facility. By use of the probabilistic model the effect of varying machine quantities in the facility can be evaluated more fully than has been possible using traditional models incorporating assumed certainty.

CHAPTER I

INTRODUCTION

Objective

The objective of this research is the development of a procedure for determination of machine and equipment requirements for a manufacturing activity which considers the random nature of affecting factors. The technique is intended to provide a better means of estimating requirements resulting from variables likely to be encountered in production than is possible using previously available techniques. The procedure will be concerned with planning of new facilities. Scheduling of these facilities will then be dependent upon dynamic conditions existing after equipment installation; scheduling is therefore beyond the scope of this research.

Justification for the Study

The matter of accurately determining the required number of machines for an activity is one which has interested industrial engineers for some time. In general, an equation has been used in this determination involving the factors of production time per unit, required quantity of production per period (hour, day, week, or year), and an equipment use factor. In the majority of studies on the subject, single levels of each factor under conditions of certainty are assumed. Where provision has been made for variation in the values of factors used in the equation, a number of discrete values, at fixed levels of activity, are assigned each of the factors. A selection of one of the discrete values for each of the factors is then made for the most probable level of activity and the number of machines determined under these selected values of certainty. In effect, this is no different than the use of a single assumed value. The failure of the model to include conditions of uncertainty in factor values and variability in activity level has considerably weakened present plant layout and facility planning techniques.

Inability to determine requirements adequately may be the most serious deficiency in planning facilities. Muther states (1, p. 42)[1], "determining the number of machines required . . . must precede any establishment of space and other requirements for machinery," and (1, p. 50) "An entire layout scheme may be limited by these machine requirements." The major plant layout consideration in recent literature has been the arrangement of machines and equipment. However, if the estimated required number of machines is in error, the arrangement of these machines will not rectify, but will only reduce, the effect of the initial error on the firm's capability and production efficiency. Errors in arrangement of a given group of machines can normally be rectified, or the adverse effects reduced, by rearrangement within the same facility area. However, under-provision of equipment during planning may require additional area, as well as partial or total rearrangement. Over-provision of equipment will result in excess capital investment, not only in the

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[1] Numbers in parentheses refer to references cited in the Bibliography.

equipment itself but in the structure housing the equipment.

The procedure includes factors of uncertainty which are present in the more significant variables affecting machine requirements. This allows the computation of the probability of under-provisioning or over-provisioning, with either's accompanying ineconomies.

Importance of the Study

The total expenditure for new plant and equipment within the United States economy, excluding agriculture, for 1961 was estimated as 34.67 billion dollars by the Council of Economic Advisors (2, p. 8). Of this total amount, manufacturing industry expenditures accounted for 13.68 billion dollars, or 39.77 per cent of the total. This manufacturing expenditure for new plant and equipment represented 2.62 per cent of the gross national income of 521.3 billion dollars for 1961 (2, p. iv).

The amount of manufacturers' new orders for machinery and equipment for 1961 was 58.5 billion dollars, or 11.22 per cent of the Gross National Product (2, p. 20). This latter amount includes replacements as well as new equipment. Although it is probable that in the normal firm available information relative to production demand when considering replacement of existing equipment has higher validity than information relative to production demand on equipment for an entirely new facility, the same basic considerations are involved in determining requirements in both situations. Unless the replacement equipment is identical in specifications and operating characteristics to that being replaced, new values of all, or a portion, of the variables affecting the number of machines required must be determined and considered. The standard methods of replacement analysis are rather fully documented (i.e., 3, 4, 5, and 6). Only Morris (6) gives significant consideration to the factors of uncertainty in analysis of the variables. Replacement investment expenditures in the national economy help to justify study of new facility requirements, since a technique derived for determination of the number of machines required for a new facility can be adapted to application under the conditions of replacement.

In spite of the significant portion of gross national product represented by investment in new or replacement equipment, very little work has been done in determination of requirements. In the literature search, only two recent periodical references relating directly to the problem were found. The first reference (7) adds the ratio of maximum demand occurring in any one period to the common factors of time, production requirements, and machine utilization. The model continues to assume the determination of the number of machines under conditions of certainty for each of the input values. Since demand tends toward maximum by inclusion of the new ratio, investment also tends toward maximum.

The second reference (8) presents the model

$$C(M) = C_1M + C_2 \int_M (m - M)\, f(m)\, dm$$

as the criterion function under a management policy of minimizing expected costs. The optimal policy is computed by setting

$$\frac{dC\ (M)}{dM} = 0$$

The result is the value of M, say M*, which satisfies the relation

$$\int_{o}^{M^*} f(m)\ dm = F\ (M^*) = \frac{C_2 - C_1}{C_2}$$

In the above,

(1) C(M) is the expected cost of a policy of providing M machines,

(2) M is the required number of machines, a random variable,

(3) C_1 is the fixed charges per machine per period, and

(4) C_2 is the cost penalty per machine period of overtime production.

In this case, C_1 and C_2 are variables dependent upon the number of machines. Morris questions the applicability of the model. In the model f(m) is not defined and unless f(m) is defined, the model accomplishes little.

Attention in current literature has been given to the related problem of assigning existing plant facilities to production requirements. In this case, the number of machines or equipment units is known and the aggregate demand on the equipment is usually assumed to be known. The usual approach is to optimize cost or profit by assignment of product quantities to specific machine models. The present research provides a higher degree of effectiveness in the determination of the quantity of new equipment to be installed initially. Application of the optimizing techniques for assignment of installed equipment can then be applied under conditions more conducive to maximizing benefits to the firm.

Limits of the Study

The procedure developed by this research may be applied to planning machine requirements for any manufacturing facility with known demand. However, if demand for a single product, or product group, is sufficient to cause continuous manufacturing to be economical a "production line" is desirable. A "production line" results in product arrangement of machines and equipment. The determination of requirements for individual machines in the line is best accomplished by use of line balancing techniques and not by the procedure developed herein. Techniques for line balancing have been developed by Salveson (22), Vazsonyi (23, pp. 302-308), and others.

Since line balancing techniques are more effective, for continuous production, or product layout, the procedure suggested in this research is intended for application when the facility has a process arrangement. Process arrangement occurs when production quantities of individual items or groups are insufficient to justify specialized production lines. A number of items are processed through machine groups according to a short range schedule. To maximize individual machine utilization under this product mix condition, machines are arranged by type (process arrangement) which minimizes total machine idle time. The procedure developed by this research provides a means of estimating the required number of machines to meet forecast demand if these machines are properly scheduled and utilized after installation. No attempt is made to include production scheduling (other than total annual requirements), in the model.

In all cases it is assumed that the correct decision as to the type of equipment to be used to perform a particular operation has been made before the use of the derived model. The model is not designed to select the best method for performing the operation. Rather, after selection of the method of performing the operation the model may be used to determine the quantity of machines or units of equipment necessary to meet forecast demand under the previously selected production procedure.

CHAPTER II

LITERATURE SEARCH

In considering the problem of determining the required number of machines for new manufacturing facilities, two related but distinct phases of the problem must be considered. The first of these is the determination of the required number of machines to complete a single assigned operation on an individual part. Second, after the determination of requirements for individual operations these requirements must be combined, or "balanced," to estimate the total requirements for a specific machine or equipment model.

Number of Machines Required by a Single Operation

Shubin and Madeheim (9, p. 46) present an elementary equation for determining machine requirements of an individual operation as

$$N = \frac{T}{60}\,\frac{P}{HC}$$

in which

N = Number of machines required.

T = Standard time for the operation in minutes.

P = Total units of production required per day of standard hours H.

H = Standard number of hours per day per machine.

C = Factor of use of equipment, taken as 0.85.

A discrete value is assigned to each of the factors and the number of machines determined under these conditions of assumed certainty.

Ireson (10, p. 22) uses a table to gather data relative to machine capacity demands of each part. Table body columnar headings provide for (1) operation time per occurrence, (2) occurrences per piece, (3) total required machine hours per period (month), and (4) the number of machines required. The quantity of pieces required per month is given as a fixed quantity in the heading of the table. The number of machines required is determined by dividing the total required machine hours per period by the number of hours available in the period. No provision is made for inclusion of equipment utilization or performance against standard.

Apple (11, p. 162) takes an approach to the problem similar to that of Shubin and Madeheim but without considering the factor of use of equipment. Instead, a "plant" efficiency factor is used. This is defined as "the average efficiency of all departments in the plant." The model relationship reduces itself to

$$\text{number of machines} = \frac{\text{required production per hour}}{\dfrac{1}{\text{standard time in hours/operation}} \times \text{plant efficiency factor}}$$

The required production per hour allows for scrap generated during production and units rejected by final inspection. This is accomplished by determining an average percentage of process loss and increasing the units produced by this amount through the use of the relationship

$$\text{required pieces per hour} = \frac{\text{good units required per hour}}{(100 - \text{per cent loss})\ .01}$$

As in the earlier references, "required good pieces per hour" is estimated by dividing total annual requirements by production hours per year.

Muther (1, p. 42) presents two simplified relationships for determining the number of machines required. These are

$$\begin{matrix}\text{number of}\\ \text{machines}\\ \text{required}\end{matrix} = \frac{\begin{matrix}\text{pieces per hour to}\\ \text{meet production}\\ \text{requirements}\end{matrix}}{\begin{matrix}\text{pieces per hour}\\ \text{from machine in}\\ \text{question}\end{matrix}} = \frac{\begin{matrix}\text{operating time per piece}\\ \text{for machine in question}\end{matrix}}{\begin{matrix}\text{time per piece to meet}\\ \text{production requirements}\end{matrix}} .$$

These relationships assume no defective work and 100 per cent machine utilization. Muther later (1, p. 339) points out that allowance must be made for capacity reducing delays and spoilage.

> Capacity delays are usually classed as machine delays and operator delays. Operator delays include personal time, idle time (avoidable delay), rest to overcome fatigue, below standard skill or effort, and miscellaneous unavoidable delays. These delays are normally reflected as a delay allowance added to the select or leveled time study figures.
>
> Machine delays (or 'machine inefficiency') involve change-over or setups, tool changes, repairs due to breakdown or wear, oiling, greasing, and cleaning, failure of utilities (power, air, water, etc.), miscellaneous interference, imperfect dispatching or shop loading, and the likes. . . . Usually machine delays are relatively difficult to measure, and considerable periods of time are necessary before representative results can be insured. However, by means of ratio delay studies or 'chronolog' analysis, satisfactory results can be obtained economically. . . . It is of prime importance that 'utilization factors' be accurately known. . . .
>
> Reject allowances . . . depend on the machinery and material. . . . To be correct, . . . the . . . engineer must determine these allowance figures for each machine. In practice, we sometimes take a short cut and use an over-all reject-and-percent-of-capacity allowance.

Under the above considerations, Muther then converts his relationship for determining the number of machines to

$$\text{number of machines} = \frac{\text{pieces required per week} + \left[\text{pieces required per week} \times \text{per cent rejects}\right]}{\text{hours per week}}$$

$$\div \left[\text{theoretical pieces per hour per machine} \times \text{per cent capacity factor}\right]$$

where the per cent capacity factor is equal to one minus operator and machine delays expressed as per cents. The theoretical pieces per hour per machine is based on the leveled time study values, thereby assuming 100 per cent operator performance. In addition, the factor [pieces required + (pieces required x per cent rejects)] gives an erroneous value for total production start quantities since per cent reject is related to pieces started rather than good product.

Muther (1, p. 48) is the only one of the above references which presents a formalized method for establishing set-up allowances to be used in determination of the utilization factor. Using fixed levels for each of the influencing factors, Muther uses the following relationship:

$$\text{set-up allowance per unit} = \frac{\text{total set-up time per machine} \times \text{number of manufacturing lots per year}}{\text{gross annual volume of production}}$$

In each of the above references, no consideration is given to variations in demand for the product. In each case a forecast is made of annual product requirements. This annual demand is then divided by the number of basic periods (normally hours) to arrive at average required production per basic period.

In a more recent article, Johnston (7, pp. 229-231) uses a demand ratio in determining "the required pieces of equipment to meet any sales forecast." Johnston presents the formulation,

$$C = \frac{T}{S \times P}$$

and

$$N = \frac{F \times D}{C}$$

or

$$N = \frac{F \times D \times S \times P}{T}$$

where

C = The capacity of each piece of equipment, in units per period.

T = The available production time (after the deduction of permitted meal-breaks, tea-breaks, and rest periods) per period in minutes.

S = The standard time to produce an average unit, in minutes.

P = A factor representing the expected performance against standard (taking into account production time lost by machine breakdown, material and supply shortages, absenteeism, and general inefficiency).

N = The number of pieces of equipment.

F = The sales forecast in terms of average units per period.

D = The ratio of maximum demand occurring in any one period of the year to the average demand per period (to insure that production facilities are available to meet reasonable short-term requirements above the average level).

Johnston states, "The period used for calculation purposes should be the period used for production scheduling--usually one week, but in some factories a month or more."

In effect, Johnston's formula provides equipment based upon maximum demand in any one period during the year. During other periods, excess capacity to demand exists. Furthermore, no consideration is given to complete or partial leveling of production. Leveling of production increases inventory investment while reducing machine investment. An optimizing policy should be based upon combined results. Johnston recognizes the factor of variable demand but resolves it by providing maximum required machine capacity and, thereby, maximum machine investment. In his formula Johnston makes no provision, as do the earlier studies, for losses due to scrap or inspection rejects. Furthermore, Johnson's use of S x P for average time per unit is in error. Average time per unit = S/P, since as performance improves average time is reduced. Therefore the corrected equation is C = TP/S.

The author, in an earlier work, used the basic formula of Shubin and Madeheim for determination of the required number of machines (12, p. 57). In this work attention was given to the factors affecting the production required (P) and the use factor (C). The following discussion relative to (P) is quoted (12, pp. 60-62):

> Production required (P), on the other hand, will vary considerably due primarily to two factors: (1) each preceding operation must produce quantities which will become nonrecoverable bad work in succeeding operations and (2) the quality level which can be maintained will vary with the equipment, material, tolerances, method of operation, and method of control involved for each operation. The total production required is equal to the required good product plus bad work, or expressed symbolically, $P = (P_G + P_{BW})$, where P_G represents required good product and P_{BW} represents nonrecoverable bad work. Time and capacity are spent on bad work as well as good work. The problem then is to determine the good product requirement and expected bad work for each operation.
>
> The required good product for the final operation (O_n) is equal to expected sales requirements. The good product requirements for the next-to-last operation (O_n - 1) is equal to the good product for O_n plus the expected nonrecoverable bad work generated by O_n. The nonrecoverable bad work is designated most often as scrap (although in some cases as rejects, seconds, or other lower quality reworkable for sale). Products may be nonrecoverable as good product although not to be discarded as scrap. $P_G(O_n\text{-}1)$ then is equal to
>
> $P_{G(O_n)} + P_{BW(O_n)}$ and $P_{G(O_{n-2})} = P_{G(O_{n-1})} + P_{BW(O_{n-1})}$,
>
> and so on. If by a market analysis or sales forecast $P_{G(O_n)}$ is established, P for each operation can be calculated if P_{BW} for that operation is determined.
>
> In addition to nonrecoverable bad work most, if not all, industry has additional bad work which by rework can be recovered as good quality product. This portion of bad work does not affect our determination of preceding operation good-product requirements above, but it does affect the total production required of each operation which may create rework conditions. Time must be spent to rework these items and additional equipment must be provided. The time required to rework an item may be greater than the time required to perform the operation originally, due primarily to each rework piece having individually peculiar requirements. This requires analysis by the operator doing the rework as well as the quality control or process engineers who may specify the

rework. Due to these individually peculiar requirements, many industries find it best to establish separate rework areas or departments. Others may schedule it during idle machine periods. The choice depends primarily upon cost and control factors present in the operation. In either case equipment excessive to first-run requirements is necessary and may be determined separately using times applicable to this type of operation. To differentiate the rework load, it should be determined by using the formula

$$N' = \frac{T'}{60}\,\frac{P'}{HC}$$

where N' = number of machines required for rework; T' = time assigned for rework; and P' = number of pieces submitted for rework. In the basic machine-requirement determination this amount will be distributed between P_G and P_{BW} in that after rework a portion will become usable and the remainder will become scrap.

If the decision is to perform rework operations within the production areas, N' should be added to N to determine total machine requirements for the operation. If a separate rework area is to be used, N' is the requirement of that area.

To properly estimate values of P_{BW} and P', historical data from similar operations are necessary. The conditions under which the data were gathered should be analyzed to determine probable effects of variations between those conditions and the conditions being created under the new layout. If quality control records have been maintained in the past, not only will it be possible to determine average bad work, scrap, and rework, but statistical control limits may also be available. If control limits were used or can be established and an analysis of reject causes prepared, an attempt should be made under the new process to eliminate or reduce the major sources of the causes for rejects. If it is probable these causes will be reduced, the average rejects to be expected can then be adjusted accordingly by re-evaluating the historical data and transferring the now controllable rejects to good product expectations.

In discussing the use factor (12, pp. 63-64), it is pointed out that:

> The use factor, or ratio of probable maximum equipment availability to total production hours, will be dependent upon the nature or design of the equipment being considered, continuity of equipment use, and maintenance policy. . . . Consideration must also be given for the ratio of setup time to line assigned

time. The apparent use factor resulting from earlier considerations (the three above) must be reduced. Therefore, actual available time = apparent available time ÷ [1 + (setup time to available use time ratio)].

In most of the above studies, mention is made of the necessity for analysis of and the distributional nature of the data affecting the determination of the number of machines required. However, there is no attempt in any instance to establish a detailed model or technique for accomplishing this analysis. Rather, the method to be used in analyzing raw data in arriving at values to be used in the basic equation under conditions of certainty is left to the discretion of the reader. Mention is made of ratio delay, simulation, Monte Carlo, and "chronolog" analysis, but methods for using these techniques are not presented. Furthermore, no instances were noted in the literature search of work which had been directly related to this problem.

Morris (8 and 6, pp. 223-224) does recognize that demand, performance time, and the effectiveness (utilization) factor have probability distribution characteristics. Morris indicates an expression for the probability distribution of the number of machines required during a period $f(m)$, based upon combining distributions of the influencing factors. However, he does not indicate the forms of any of the distributions. Instead, after a general representation of the probability distribution of the actual number of machines required in the period, he concludes his discussion with:

> If the . . . distributions are taken to be of analytically known form, then there are analytic methods for obtaining the distribution of m. If, however, the forms are not obtained, one might use the Monte Carlo method for obtaining an estimate of the distribution of m. This step can be a very difficult one in actual practice.

Therefore, Morris has not resolved the problem but merely placed the generally recognized problem in a more rigorous mathematical generalization.

To accomplish this more rigorous generalization, Morris writes the basic formula of Shubin and Madeheim under conditions of certainty in the form

$$\overline{M} = \sum_i \sum_j \frac{\overline{T}_{ij}\ \overline{D}_j}{\overline{E}\ H}$$

where

$\overline{M}$ = The mean number of machines required by the department.

$\overline{T}_{ij}$ = Mean performance time for operation i on product j, measured in hours per unit products.

$\overline{D}_j$ = Mean demand for product j measured in units per production period.

H = Number of hours in a production period.

$\overline{E}$ = Mean effectiveness factor--a decimal taking into account the usual personal allowances, machine downtime, material shortages, scrapped production, etc.

It will be noted that if the summation signs are removed, the formula provides the mean requirement of an individual operation in the same manner as that of Shubin and Madeheim. Summation of individual operation average requirements provides average total requirements for the machine model being considered. As will be pointed out later under total machine model determination, this is an inadequate method for determining total requirements.

Working from his formula for conditions of certainty, Morris states:

> In order to exhibit an initial model, let us assume that performance times, demands, and effectiveness factors are random variables which may be described by probability distributions. Thus, let
>
> $f(T_{ij})$ = Probability distribution of T_{ij}.
>
> $g(D_j)$ = Probability distribution of D_j.
>
> $h(E)$ = Probability distribution of E.
>
> We assume H is fixed by policy and not subject to important variation. . . . Clearly, if these variables are taken to be random variables, then the required number of machines, m, will itself be a random variable.
>
> Thus
>
> $$f(m) = \sigma\,[f(T_{ij}),\ g(D_j),\ h(E)]$$

As pointed out earlier, the nature of these distributions is not discussed. As a result, no significant improvement in determination techniques is made.

Morris does, however, develop a decision model under a management policy of minimized expected costs (8, pp. 35-39; and 6, pp. 225-226). To accomplish this, the criterion function is written as

$$C(M) = C_1 M + C_2 \int_M^{\infty} (m - M)\, f(m)\, dm$$

in which

C(M) = The expected cost of a policy of providing M machines.

C_1 = Fixed charges per machine per period.

C_2 = Cost penalty (excess over regular time) per machine period of overtime production.

$f(m)$ = Probability distribution of actual number of machines required in a period.

The optimum policy is then computed by setting

$$\frac{dC(M)}{d(M)} = 0.$$

The result is a value of M taken as M*, which satisfies the relation

$$\int_{0}^{M^*} f(m)\, dm = F(M^*) = \frac{C_2 - C_1}{C_2}$$

where M is assumed as a continuous variable. Under this model, it is assumed that costs are linear. In practice the ratio may not be constant but vary with the number of machines and demand against the machines.

Morris does not represent the above model as being generally applicable, pointing out that whether the model fits the real decision depends on a number of assumptions. He does not list these assumptions, but they are primarily involved in determining C_1, C_2, and $f(m)$. He closes his discussion of the model by stating "fitting the model is, of course, a tremendous tactical problem."

Morris also develops a model (8, p. 360; and 6, p. 227) for the probability of having to produce D and k pieces in order to obtain D good pieces. If p is the mean scrap rate, the model is

$$\phi(D,\ k,\ p) = \frac{(D + k - 1)!}{k!\ (D - 1)!}\ p^{k}\ (1 - p)^{D}.$$

Morris points out the joint probability distribution model may become quite complicated, and although "experience seems to indicate that it is worthwhile introducing probability distributions, . . . very complicated distributions may go too far."

In his discussion of the problem, Morris develops a set of curves representing the general nature of costs of errors in planning based on the assumed certainty approach (8, pp. 260-261; and 6, pp. 228-230). These are reproduced as Figs. 1, 2, and 3. Fig. 1 indicates the relationship between C_1 and C_2 as defined in Morris' cost optimizing model and the optimum number of machines. As C_1 becomes smaller relative to C_2, the optimum number of machines is increased. Deviation from the optimum-number-of-machines curve will result in increased cost. Therefore, if costs C_1 and C_2 are disregarded and the number of machines is determined based upon average production requirements, suboptimization results. The degree of suboptimization, and thereby amount of increased cost, is dependent upon the degree to which the values of C_1 and C_2, which in effect are assumed by average production requirements, deviate from the true values of C_1 and C_2.

In Fig. 2, the effect of failure to recognize the variation in the optimum number of machines is illustrated. As the variation increases, the optimum number of required machines also increases. Only when variation of requirements is zero do the machines for average requirements equal the optimum number of machines. When variation exists, the average machine requirement is suboptimal. As variation increases, the degree of suboptimization also increases. In Fig. 3, the relative total expected costs of the two methods are plotted as functions of the variation in m. The vertical distance between the two curves represents the increased cost due to a policy of calculating machine requirements based upon average requirements.

Morris does not indicate methods for determination of values on any of the above curves. However, his presentation is the only one found in the literature search which attempts to begin formalizing an approach under conditions of uncertainty.

Fetter (13, pp. 13-7, 13-8) considers the problem of determining the number of units to start through a process when the number of required good units is known, and there is a probability distribution associated with the fraction of defective items from the process. He accomplished this by tabulating the frequency with which a given ratio of pieces started to good pieces resulting has occurred in the past. These results are then used to plot a probability distribution in the manner of Fig. 4. The ratio of units started to good units resulting is the variable "x." If the variable cost of each piece started is "V," and of an additional setup required, is "S," the expected cost of adding the i^{th} unit to the production order is $P(X_i)V$. An additional setup cost will be incurred if the order size "i" is not large enough to

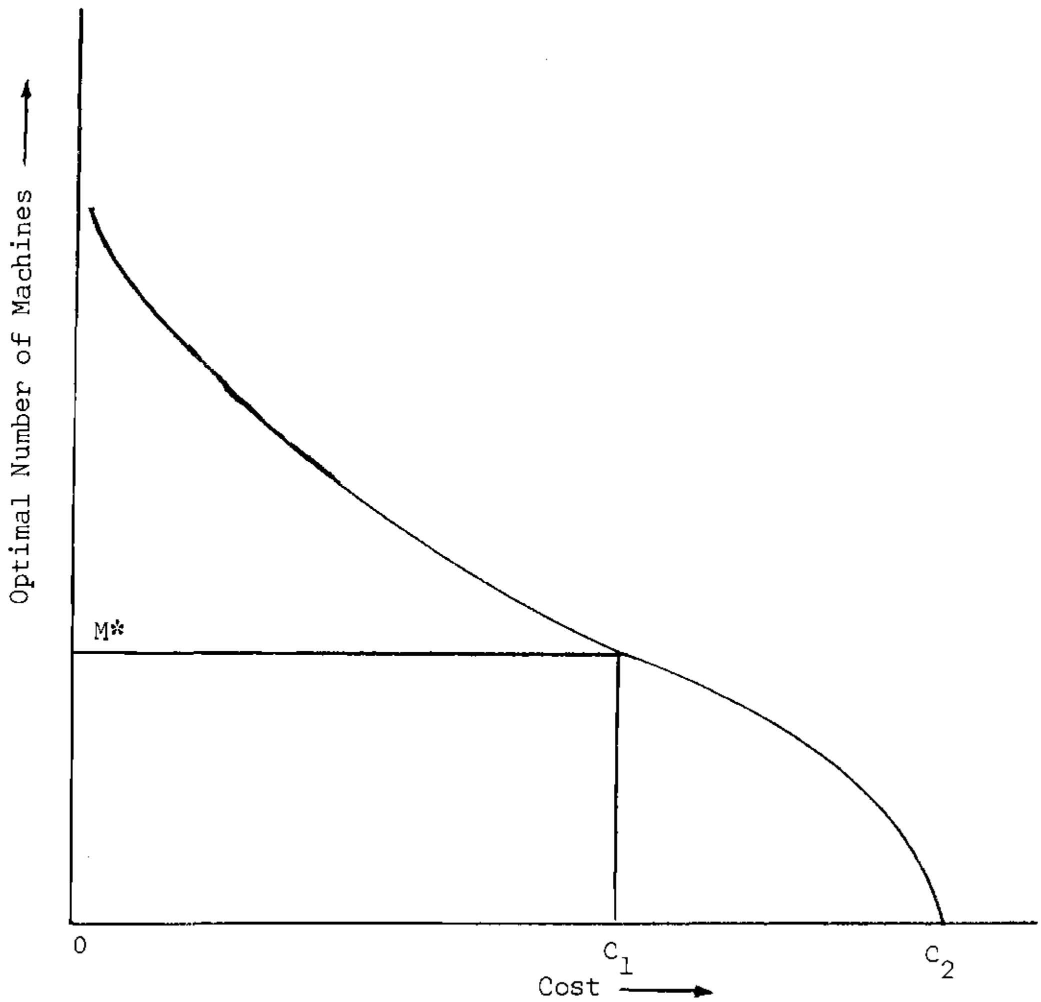

Figure 1. Optimum Number of Machines vs. Ratio of C_1 to C_2

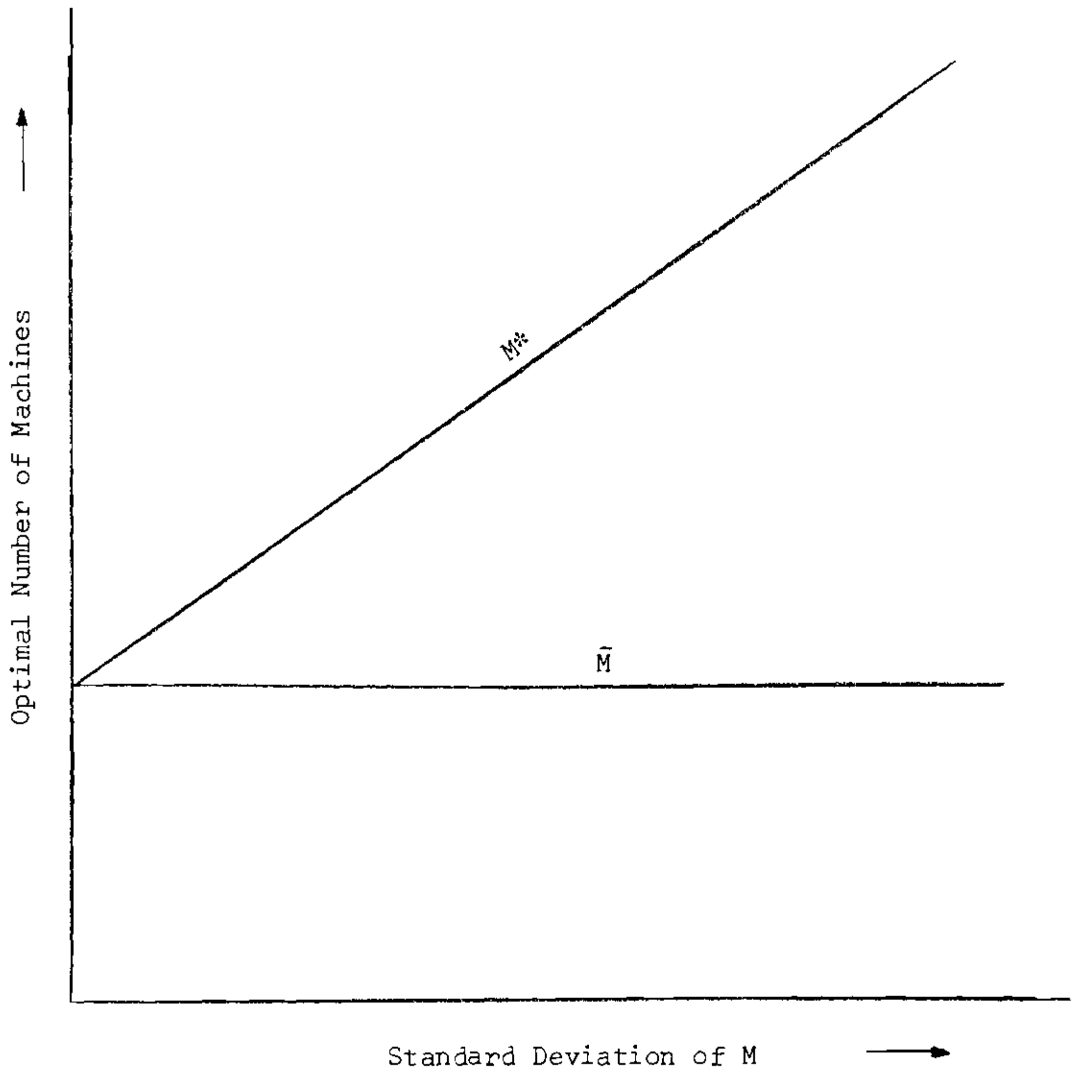

Figure 2. Result of Failing to Recognize Variation in the Required Number of Machines.

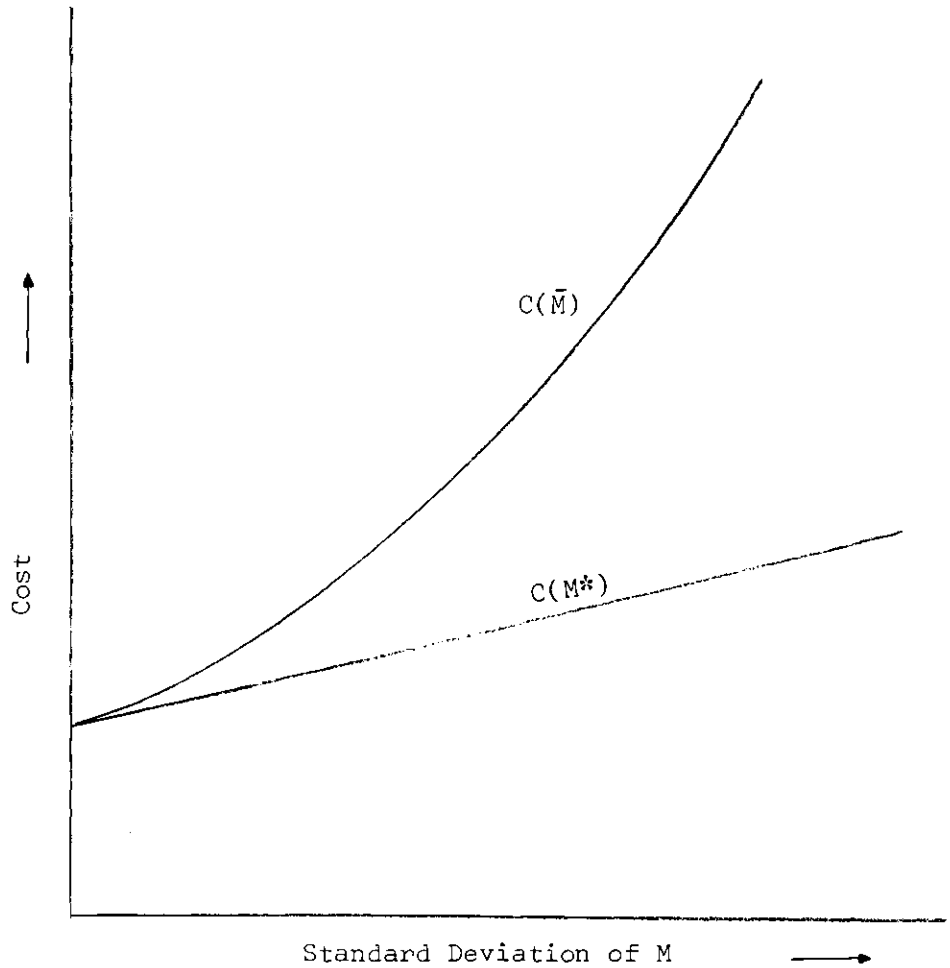

Figure 3. Cost Effect of Variation Present in Machine Requirements.

satisfy good product requirements. The probability that the $(i + 1)^{th}$ piece will be required is $[P(X_{i+1}) - P(X_i)]$. Thus the minimum cost point is where

$$V[P(X_i)] = S[P(X_{i+1}) - P(X_i)]$$

and

$$\frac{P(X_{i+1})}{P(X_i)} = \frac{V + S}{S} .$$

The decision rule under this model is to select an order size, "i," in order that the ratio

$$P(X_{i+1}) / P(X_i) = (V + S) / S.$$

Total Requirements for a Machine Model

Except for Morris' formula for determination of machine requirements under certainty, consideration to this point has been given to determination of machine requirements demanded by a single operation. Morris' formula

$$\overline{M} = \sum_i \sum_j \frac{\overline{T}_{ij}\ \overline{D}_j}{\overline{E}\ H}$$

provides for the average total requirements of a particular machine model

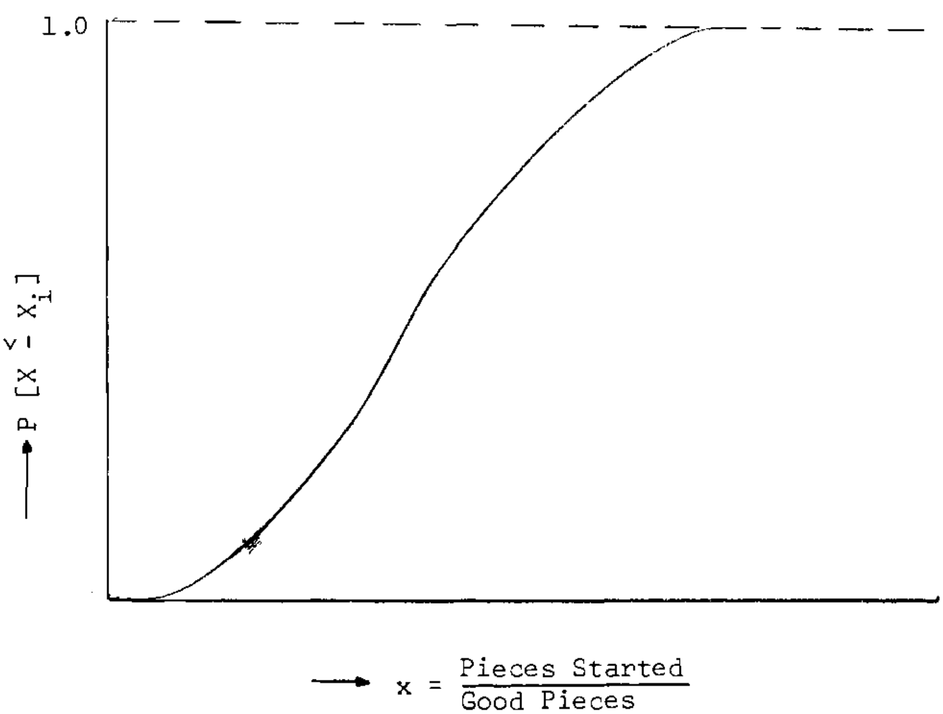

Figure 4. Probability Distribution of the Ratio Pieces Started to Good Pieces.

by summing average requirements for that machine model across all operations (i) and products (j). In arriving at total requirements in this manner, the problems of scheduling the product mix across the machine group is disregarded. The inherent assumption in this approach is that by summation of individual requirements sufficient capacity is provided that a schedule is feasible and can be determined after equipment installation. It is also assumed that excessive requirements during any period must be provided through overtime operation. Morris recognizes this problem, since in his model for the optimal policy under uncertainty the resulting optimal number of machines is dependent upon the relative value of the fixed costs and the overtime cost penalty per machine period. Morris, in his model under uncertainty, appears primarily concerned with requirements for an individual operation. Pursuing this approach for determination of total requirements will involve a joint probability distribution function for the use factor, since variation is generated by both frequency and nature of setups. One of the major problems in determination of machine requirements will involve the development of a method for handling the changes in the use factor as the number of operations crossing the machine is varied. An interdependency exists between individual operation requirements and total requirements due to this variation generated in the use factor.

No satisfactory means of handling the problem of total machine model requirements was discovered during the literature search. Muther (1, p. 48) uses a tabulation of average requirements of individual operations which are summarized to arrive at total machine model requirements. The significant portions of his tabulation approach are reproduced in Fig. 5. This, in effect, is Morris' model under certainty.

Moore (14, pp. 131-132) and Ireson (10, pp. 21-23) determine total machine requirements for a process layout by summing the monthly machine capacities required for individual operations on that machine model. The individual operation required capacity is the sum of setup time and average production hours demanded per month by the operation. Setup time is determined by multiplying the average time per setup by the estimated number of setups required per month. The required number of machines is then obtained by dividing the total capacity in hours required of the machine model per month by the number of operating hours per month. In effect, this provides requirements under conditions of average production time per piece, average time per setup, and average setups per month. No allowance is made for variability in the three factors.

In the above references, the required number of machines is taken as the next larger whole number above the value of summed operation requirements when this sum is a mixed number. Apple (11, pp. 163-164) points out that when the resulting sum is a mixed number there continues to exist a question as to the number of machines to provide.

Such decisions are largely a matter of judgment based on factors such as:

> (1) How much of the man cycle is man-controlled and might yield more pieces per hour with more favorable conditions?
>
> . . .
>
> (2) Can the method be changed to reduce the standard time?

General Class of Equipment			Presses						
Equip. is presently located in dept.			705	732	705	705	701		
Product Design No. W 33G Production Year 195X Production Forecast 50,000 (15/Hr.) Signature J.C.L. Date Originated Sheet 1 of 4			126 Ton Niagara #37818& 3782	105 Ton V. & O. #5388 & 5412	70 Ton Bliss #5453	70 Ton H. & W. #3975	60 Ton York #3007-'11-'19-'55	50 Ton V. & O. #4341	
	Avail.	No. Machs on floor	2	2	1	1	4	1	(1)
		Min/Hr. @ 100% Ut.	120	120	60	60	240	60	(2)
		Utilization Factor	95	90	75	75	95	955	(3)
		Min/Hr. @ % Ut.	114	108	45	45	228	57	(4)
S/U Appn.1-700	Demand	Unit Ld. Inc. Setup	2.75	4.90	--	1.00	6.20	--	(5)
Product Name		Antic. Makeout	100	90	110	110	95	100	(6)
		Unit Ld. Adj.to M.O.	2.75	5.44	--	0.91	6.53	--	(7)
1/3 Window Unit	15	Program Ld. Min/Hr.	41.2	81.5	--	13.7	97.8	--	(8)
1/2 Window Un	15	Program Ld. Min/Hr.	52.4	85.0	--	13.7	105.2	--	(8)
3/4 Window Un	20	Program Ld. Min/Hr.	40.2	47.0	24.1	--	62.7	10.0	(8)
3/4 Console LC	10	Program Ld. Min/Hr.	27.0	78.4	16.9	7.4	51.0	--	(8)
1 1/2 Console	7	Program Ld. Min/Hr.	105.0	20.2	41.0	--	4.5	16.0	
#450 A.I.M.	2	Program Ld. Min/Hr.	39	16	75.0	14.0	46.0	2.1	
#550 B.W.	4	Program Ld. Min/Hr.	75.1	19	102	20	79	40.1	
Grand Summary Sel. Min/Hr.			338.7	315.6	259.0	55.1	348.4	68.2	
Operator Delay Allowance -- %			15	15	12	12	15	10	
Total Allowed Time -- Min/Hr.			390	362	290	62	400	75	
(Similar Mach. -- Accum. Demand.						352			
(Similar Mach. -- Accum. Avail.						90			
Gross No. Machines Indicated			6.9	6.7		7.8	7	1.3	

Fig. 5. Muther's Tabular Summary of Machine Requirements

> (3) Is overtime cheaper than an additional machine?
>
> (4) Would a breakdown of a single machine shut down the line?
>
> It would be much harder for one machine to turn out production for 1.25 machines than it would be for 6 to turn out production for 6.42. . . . Actual decisions in such cases are made on the basis of past experience and detailed knowledge of plant conditions.

Models for Allied Problems

In a study made by Schiller and Lavin (15, pp. 231-234), a determination of requirements for warehouse dock facilities was made. In this instance, a survey was made of truck arrivals at three warehouses. The distribution of empirical data was found to be approximated by a Poisson distribution. It was further found that unloading and servicing time was closely approximated by the negative exponential distribution. These approximating distributions of the problem were used to simulate conditions using a Monte Carlo procedure programmed on a Datatron computer. Repeated simulation over a range of discrete numbers of docks permitted a least-cost-capacity decision. Details of the Monte Carlo program and results were not presented.

The approach of Schiller and Lavin is probably applicable to certain machine problems. However, in applying the technique, Schiller and Lavin utilized three major conditions which would likely be complicated by the machine problem:

1. Dock loading could be studied under actual operating conditions in order to develop empirical distributions of system conditions. It was possible to collect the data to derive distributions within a seven-week period.

2. Only two classes of trucks (products in the machine analysis) were being serviced.

3. It was not necessary to incorporate a factor similar to the use factor involved in machine study. The presence of a use factor would add complexity in establishing operating capacity constraints for a discrete number of docks in the Monte Carlo program.

Another quantitative technique developed for a problem similar to the machine requirement problem is that of Morris (16, pp. 48-53) for determination of design capacity for a random flow pattern of materials handling. Morris considers the problem of assigning vehicles for handling between a warehouse and a plant. No restrictions as to the number of vehicles available is considered. The total cost function,

$$C(k) = C_o k + C_1 \sum_{D_1=kT+1}^{\infty} (D_1-kT)f(D_1) + C_2 \sum_{D_2=kT+1} (D_2-kT)g(D_2),$$

is minimized by seeking the minimizing value of k. The "k" value is discrete, and the minimizing value, k', must satisfy two conditions:

$$(1) \quad C(k' + 1) - C(k') \geq 0$$

$$(2) \quad C(k' - 1) - C(k') \geq 0$$

For condition (1), Morris obtains:

$$C(k'+1)-C(k')=C_o(k'+1)+C_1 \sum_{D_1=(k'+1)T}^{\infty} [D_1-(k'+1)T]f(D_1)$$

$$+ C_2 \sum_{D_2=(k'+1)T}^{\infty} [D_2-(k'+1)T]g(D_2)$$

$$- C_o k' - C_1 \sum_{D_1=k'T}^{\infty} (D_1-k'T) f(D_1)$$

$$- C_2 \sum_{D_2=k'T}^{\infty} (D_2-k'T) g(D_2) \geq 0$$

where

D_1 = Loads to be moved from the warehouse to the plant.

D_2 = Loads to be moved from the plant to the warehouse.

$f(D_1)$ = Probability distribution of D_1.

$g(D_2)$ = Probability distribution of D_2.

k = Number of vehicles assigned for a scheduling period.

T = Number of round trips each vehicle can make within the scheduling period.

C_o = Total cost (fixed plus variable).

C_1 = The cost of failing to move a load from the warehouse to the plant.

C_2 = The cost of failing to move a load from the plant to the warehouse.

kT = Number of round trips which <u>will</u> be made during the period.

Morris at this point simplifies the solution by assuming T = 1, and $C_1 = C_2$. This permits writing the above relationship for condition (1) as:

$$\frac{2C_1 - C_o}{C_1} \leq F(k') + G(k').$$

By using condition (2) in the same manner, Morris states the result is:

$$\frac{2C_1 - C_o}{C_1} \geq F(k' - 1) + G(k' - 1).$$

The value of k' which satisfies the two inequalities gives the required number of vehicles.

In effect, Morris has over-simplified the problem before establishing his decision model. In addition, assumptions are made that (1) all material is ready to be moved at the beginning of the scheduling period, and (2) vehicles will move either full or empty resulting in a load factor of either zero or one. Assumption (1) results in no delay waiting for work. Assumption (2) permits determination of a utilization factor by use of the formula

$$\frac{\sum_{D_1=0}^{kT} D_1 f(D_1) + \sum_{D_2}^{kT} D_2 g(D_2) + \sum_{D_1=kT+1} kTf(D_1) + \sum_{D_2=kT+1} kTg(D_2)}{2kT}$$

but this does not allow variation in kT, and therefore, no allowance for equipment breakdown or other delay.

As Morris himself points out (16, p. 54),

> This approach usually misses the mark in at least two important ways. The requests for movement do not in general arise at just the moment the materials handling system is ready to serve them. . . . In addition, it is unusually difficult to predict definitely the productivity or effective output rate of materials handling equipment.

This statement is equally applicable to production equipment.

Perhaps the greatest amount of work in allied areas has been concerned with problems generally classified as assignment problems. In general, assignment problems are concerned with the assignment of product mix to available production capacity in an established plant. The type, number, and capacity of production equipment are assumed known. The total required production for single or multiple products is then assigned to the known capacity in such a manner as to maximize a stated objective. The objective is usually profit maximization or cost minimization. The problem of assignment of production to existing capacity is normally formulated for solution by either the simplex or distribution methods of linear programming. When problems are formulated with unity rim conditions, they are designated as assignment problems (17, p. 39), but the general problem considered is not limited to this restriction.

Several references in the field of linear programming consider the problem of production assignment of known capacity. Bowman and Fetter (18, Chapter 4) solve the problem using the simplex. In this case, not only is capacity assumed known but no variability is allowed in the time required by an individual operation on the product. Bierman, Fouraker, and Jaedicke (19, Chapter 15) also use the simplex.

Reinfeld and Vogel (20) use the simplex (pp. 75-89) and the distribution method (pp. 137-142). In their simplex formulation, consideration is given to the machine utilization factor but only under conditions of certainty.

McRae (21) solves the problem geometrically (pp. 374-378) and by the simplex (pp. 378-386). In transferring from the geometric to the simplex, for an illustrative example, McRae increases the variety of product mix and machine capacity. The problem in both cases is solved under certainty with changes in the discrete levels of production requirements and machine capacities. As with other solutions by general linear programming techniques, no variability is incorporated in requirements or capacity within the individual formulations.

In using the linear programming approach to the determination of the number of machines required, a different solution is possible for each capacity change represented by increasing or decreasing the assumed quantity of an individual machine model. This would necessitate

$$\sum_{j=1}^{N} \sum_{i=1}^{n_j} a_i N_j$$

solutions, where N_j = the j^{th} machine model, a_i = an assumed number of machines for Model N_j, and n_j is the number of discrete values of a_i investigated for N_j. The minimum total cost, and thereby, an optimum capacity could be determined by comparing total cost for each of the

$$\sum_{j=1}^{N} \sum_{i=1}^{n_j} a_i N_j$$

solutions. If the number of products were M, each conditional solution would involve solving an M x N matrix programming model. The cost of such an approach could be prohibitive. Furthermore, no allowance would be made for variation in productive time per piece, the machine utilization factor, or the ratio of good to total production quantities. Due to these factors being considered under certainty and the magnitude of the possible cost, the general linear programming approach used for assignment of production to machine models in an established plant under conditions of known machine capacity and product demand is not considered adequate for planning a new facility.

CHAPTER III

THE INITIAL MODEL

As pointed out in Chapter II, previously proposed methods for determining machine requirements for a single operation in a new manufacturing facility use variations of the basic formulation of Shubin and Madeheim

$$N = \frac{T}{60} \frac{P}{HC} . \qquad (3.1)$$

By expressing standard time per operation, T, in hours rather than minutes and rearranging,

$$N = \frac{T}{H} \frac{P}{C} . \qquad (3.2)$$

H, the standard number of hours per day per machine, is equivalent to the actual number of hours per day multiplied by performance against standard, or

$$H = H_a A \qquad (3.3)$$

where

H_a is the actual hours per production day per machine.

A is the performance against standard expressed as a decimal equivalent of the ratio of standard hours earned per day to actual hours per day. Dimensionally,

$$\frac{\text{standard hours/day}}{\text{machine}} = \left[\frac{\text{actual hours/day}}{\text{machine}}\right] \cdot \left[\frac{\text{standard hours/day}}{\text{actual hours/day}}\right]$$

Reducing the right side of the equation,

$$\frac{\text{standard hours/day}}{\text{machine}} = \frac{\text{standard hours/day}}{\text{machine}}$$

Actual hours per day may be assumed constant since this is established by management decree and,

$$N = \left[\frac{1}{H_a}\right] \frac{T}{A} \frac{P}{C}$$

But, T, standard time per operation, is also assigned as a constant value by normal work measurement systems. Therefore, T/H_a is a constant. Rearranging and separating the constants from the variable factors,

$$N = \left[\frac{T}{H_a}\right] \left[\frac{P}{AC}\right].$$

Dimensionally,

T = Standard hours/operation cycle.

$H_a = \dfrac{\text{Actual hours/day}}{\text{machine}}$.

P = Operation cycles/day.

$$A = \frac{\text{Standard hours/day}}{\text{actual hours/day}}.$$

$$C = \frac{\text{Actual hours per day of equipment use/machine}}{\text{actual hours per day/machine}}$$

a dimensionless ratio.

and

$$N = \left[\frac{\text{standard hours/operation}}{\text{actual hours/day/machine}}\right] \cdot \frac{\text{operations/day}}{\left[\frac{\text{standard hours/day}}{\text{actual hours/day}}\right]}$$

$$= \text{Machine(s)}$$

which since only a unique operation is involved, N = machines per operation. Therefore, the relationship is dimensionally correct and N, the number of machines per operation, is a function of the variables P, A, and C. Note that in the dimensional analysis the time period of analysis, day, cancels and does not affect the results. Therefore, as long as the same time period is used to measure H_a and P, the length of the period does not affect the solution of the model.

To facilitate symbolism for later derivations the initial model is defined as

$$N = \frac{T}{H_a} \cdot \frac{P}{AC} \qquad (3.4)$$

in which

N = Number of machines per operation.

T = Standard hours per operation cycle.

H_a = Actual hours per period per machine.

P = Number of operation cycles per period.

A = Performance against standard.

C = Factor of use of equipment.

Since T/H_a is constant,

$$N = \frac{T}{H_a} \cdot f(A, P, C).$$

The problem is now reduced to definition of the characteristics of the three variables, A, P, and C. In Chapters IV, V, and VI, these variables will be treated individually. In Chapters VII, VIII and IX, the joint function, f(A, P, C), will be established and the expression for N defined.

The above discussion and the next four chapters will be concerned with the number of machines required to perform a single operation. Later chapters will develop methods for combining individual operation requirements to obtain total requirements for a machine model. An operation in this discussion is defined as the combination of successive work elements required in the established process for producing a part which are assigned to a single man-machine combination. It is that portion of the production process which would normally be designated as a single operational step on a production routing, operation, or assembly sheet.

CHAPTER IV

PERFORMANCE EVALUATION

Performance has been defined as a decimal equivalent of the ratio of standard hours earned to the actual hours worked to earn those standard hours. This ratio can be established only after a standard of performance measured in, or convertible to, standard hours per piece has been established. The standard of performance is established as standard time per operation.

Note that performance in this case has no direct relationship to performance rating as used in making stop watch time studies. Performance rating is the subjective evaluation by the time study observer of the performance of the observed operator against a normal performance as conceived by the observer during a single operation cycle. Much of the criticism of industrial engineering in general and time study in particular has originated from this subjective evaluation against a variable normal. Predetermined time standards have tended to reduce this inconsistency but have caused new questions to arise, such as the effect of preceding and succeeding elements on the assigned time allowance for a body motion element.

Morris (6, p. 139) defines three basic types of scales. These are:

1. An ordinal scale which is simply a ranking.
2. An interval scale which has an arbitrary zero point and a constant unit of measurement.

3. A ratio scale which has an absolute zero point and a constant unit of measurement.

Time study, and its procedural element, performance rating, attempt to use a ratio scale against an absolute zero. In so doing, however, a basis for questioning the results is established. If, however, we use the standard established by time study to establish an interval scale with the time standard value as an arbitrary point on that scale, we can also establish a related arbitrary zero. We do this by designating the standard time for the operation as one standard time. Our measurement unit is now standard time which is subdivided into intervals equal to the original time measurement against an absolute zero. If the time required to perform the operation in practice is initially measured against the absolute zero scale, a point is established on the standard time scale as a result which is no longer established by the subjective judgment of the analyst. For a single cycle of the operation this is a fixed point established by measurement. The reciprocal of actual time measured on the standard time scale represents performance against standard or $T_a/T = 1/A$ in the initial model. This relationship of scales is illustrated in Figure 6.

The actual time required per operation cycle varies. The variation results from variations present in the pace of a single operator, variations in the average pace of individuals, and changes in the physical conditions relating to the operation including machine, material, or environmental factors. The actual time per cycle due to these variables will be distributed as a random variable (26, pp. 390-391). This is confirmed by A. Abruzzi, W. Gomberg, and others.

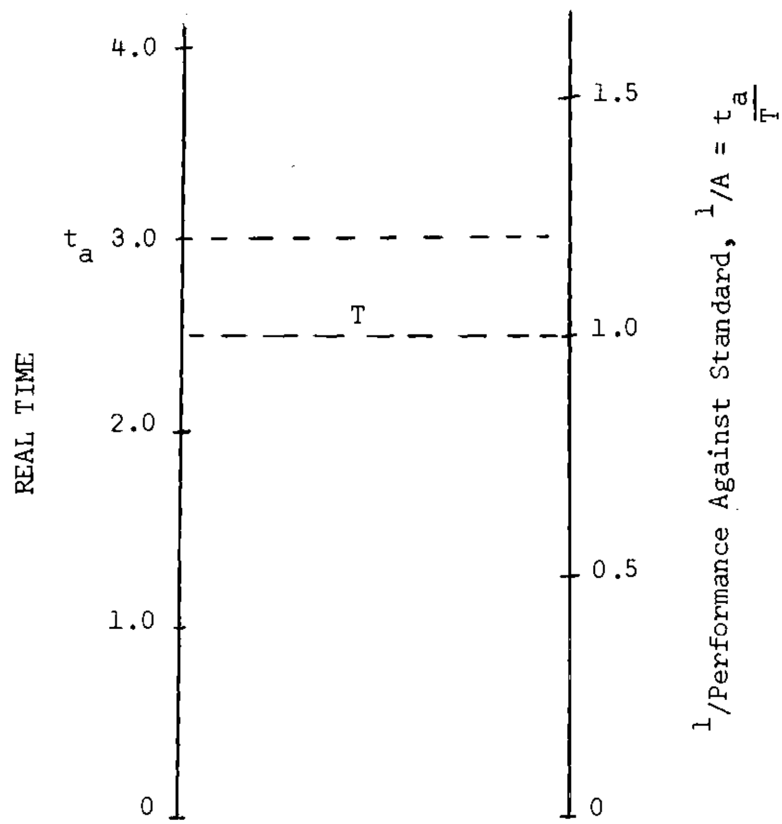

Figure 6. 1/(Performance Against Standard) Measured on a Scale With One Standard Time as the Scale Interval.

If the ratio, actual time to perform a task to task standard time, is plotted against performance measured by the standard time scale the relationship is as shown graphically in Figure 7. If the same data are plotted on logarithmic scales rather than arithmetical the result is a straight line with a negative 45-degree slope, as shown in Figure 8.

The equation of the curve plotted in Figures 7 and 8 is

$$A = \frac{1}{\bar{t}_a/T} \tag{4.1}$$

where,

$\bar{t}_a$ is the average actual time required to complete the task or operation cycle.

Rearranging,

$$\frac{\bar{t}_a}{T} = \frac{1}{A} \tag{4.2}$$

Substituting in the initial model:

$$N = \frac{T}{H}\,\frac{\bar{t}_a}{T}\,\frac{P}{C} = \frac{1}{H} \cdot \frac{\bar{t}_a P}{C} \; . \tag{4.3}$$

<u>Independence of Average Actual Time</u>

At this point the question as to the independence of $\bar{t}_a$ arises. There is little question that over the short run production demand on

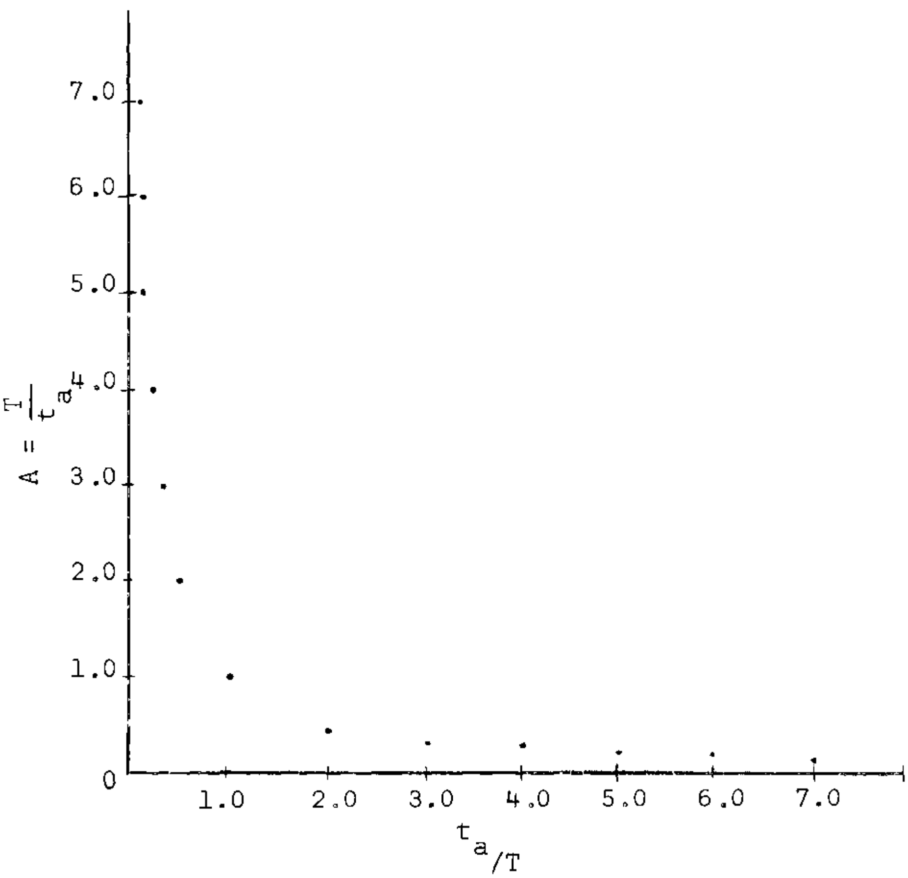

Figure 7. t_a/T vs. Performance, A.

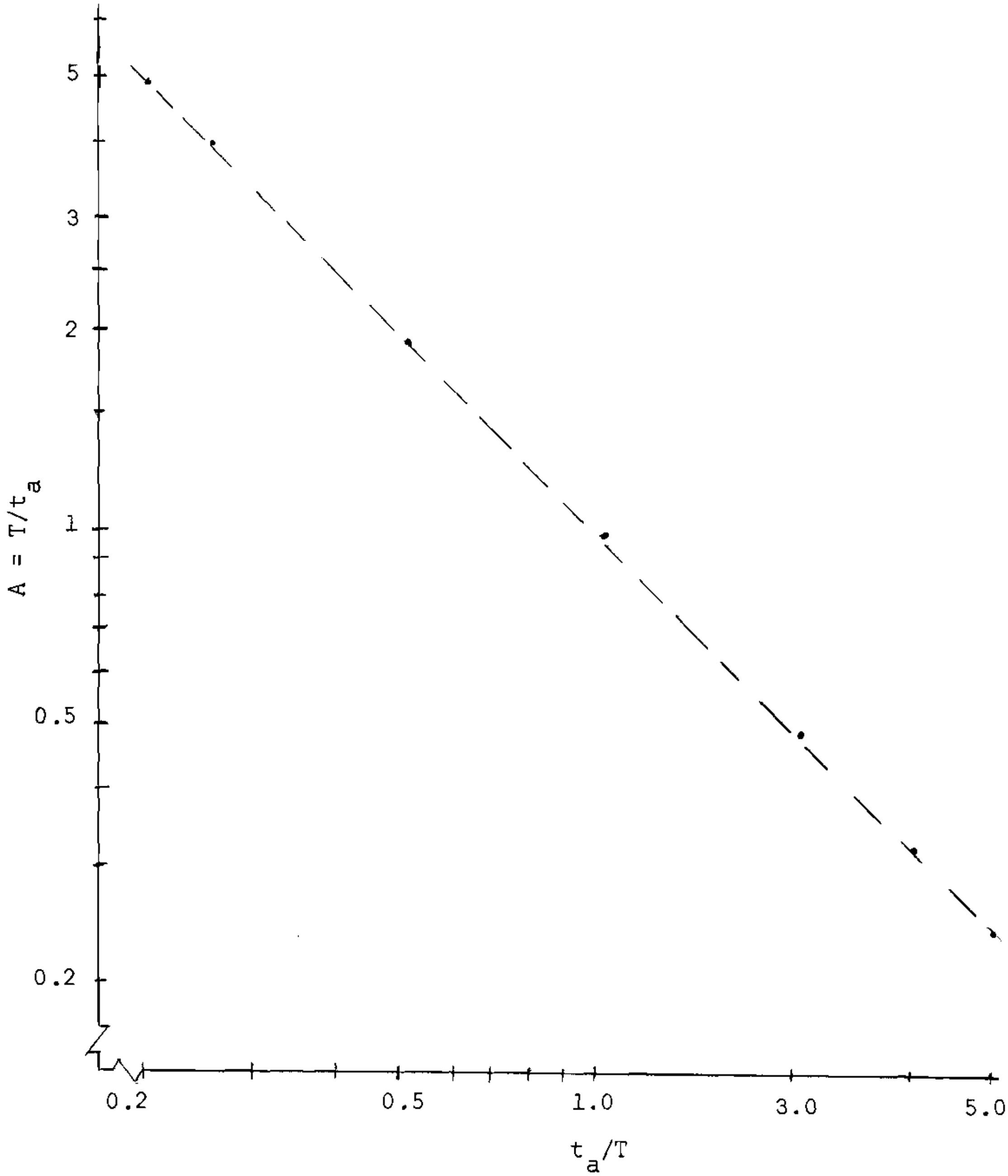

Figure 8. t_a/T Performance, A

a shop, department, or operator affects performance and average actual time. However, if load trend continues upward or downward management adjusts the labor force upward or downward When tomorrow's or next week's work load drops below a certain level workers are requested not to report. The specific times at which these reduced loads occur are not in general predictable, being dependent upon immediate general economic, sales, and production conditions. It is at least partially due to this short run fluctuation that production demands represented by sales requirements are forecast on a yearly basis. Plant physical requirements are also most often established on this yearly forecast.

Short run variations must be resolved by Production and Production Control departments. Most products have a seasonal sales pattern. Well managed companies, however, attempt to combine product mix to hold the production level at a more or less uniform level throughout the year. Since for the long run labor is adjusted to production demands, it is assumed performance and average actual time are independent of production requirements and the work load per man employed is held relatively constant by management control.

So long as management policy and practice remain constant and standards continue to be established in the same manner as in the past we can reasonably expect the frequency distribution of average actual time for next year, or the following years, to be similar to that of the past. By this argument it is assumed that the distribution of average actual time is independent of annual production requirements. This in turn results in the necessity for using as the time period of the model a production year rather than a day as used by Shubin and Madeheim.

The argument of the independence of performance is reinforced to some extent by Wall's (29) study of performance in a large job shop. Wall indicates that measurement and follow-up improve average performance and average time to a point at which average performance tends to stabilize even though short run variations of production exist. Those cases which will not permit the assumption of independence for average actual time are beyond the scope of this paper.

The Distribution of Average Actual Time

Research by Forrester (30) and Penuela (31) indicates the assumption of normal distribution for element and operation cycle times is not justified. Forrester found varying degrees of skewness and kurtosis in frequency histograms, indicating no single distribution is satisfactory for element times. Abruzzi (32) and Kennedy (33) reached the same conclusions for short cycles but Kennedy found indications of normality for longer cycles and Abruzzi concluded normality if the cycle duration exceeded 0.05 minutes. Because no general distribution for individual cycles appears reasonable the use of average actual time as the random variable in the model is necessary. The central limit theorem states, "if a population has a finite variance σ^2 and mean μ, then the distribution of the sample mean approaches the normal distribution with the variance σ^2/n and the mean μ as the sample size n increases." (34, p. 72). By this theorem the distribution of the average actual time can be approximated by the normal distribution. Furthermore, since we are not primarily concerned with parameters of the distribution of individual cycle times it is not necessary that the individual sample in terms of number of cycles timed be constant. Letting $\bar{t}_a$ be the random variable,

$$\hat{\mu}_{\bar{t}_a} = \frac{\sum_{i=1}^{N} t_i}{\sum_{i=1}^{N} n_i} \tag{4.4}$$

in which

$\hat{\mu}_{\bar{t}_a}$ is an estimate of the mean of the distribution of $\bar{t}_a$.

t_i is the total actual time of the i-th sample.

n_i is the number of operation cycles in the i-th sample.

N is the number of samples.

$\hat{\mu}_{\bar{t}_a}$ is an estimate of the mean of the distribution of t_a

and the variance of $\bar{t}_a$ is estimated by $s^2_{\bar{t}a}$.

$$s_{\bar{t}a}^{\;2} = \frac{\sum_{i=1}^{N} (\bar{t}_{a_i} - \hat{\mu}_{\bar{t}a})^2}{N - 1} \tag{4.5}$$

The permissive use of variable sample size may be of value in practice. A convenient sample size is the production per time period or units per lot. Neither of these are constants. Since we know variation exists in operation cycle time, variation must also exist in production per time period. Furthermore, if business and production control records are kept in reference to number of production lots processed through operations, then a variation in lot size processed will exist for all operations other than the first if constant lot sizes are started through the production process at the first operation. If the started lot size is variable, lot size at all operations including the first will be variable, resulting in variable sample sizes when each lot represents a sample. Lot size variation at all operations following the first results from variable defective quantities being generated and removed from the lot at each operation. At any operation lot size, and thereby sample size, will depend upon the actual number of defectives removed from the started lot during the preceding operations.

To determine the cycle time of an individual operation by work sampling it will be necessary to time the next cycle after the observation is started. It is highly likely that this will permit bias to enter because of the fact that the operator knows that he is being timed. Due to this bias the applicability of work sampling is questionable. It is recommended that if sampling is necessary to establish average times an attempt be made to use automatic cycle timing devices on the machines, cameras, or some other method which will eliminate or reduce to reasonable levels the likelihood of operator bias effects in the sample.

CHAPTER V

DETERMINATION OF TOTAL PRODUCTION REQUIREMENTS FOR AN OPERATION

The determination of total production requirements for an operation is dependent upon two major factors. First is the total sales requirements for the part or product for whose production the operation is required. Second is the expected losses of quantity resulting from the operation under consideration and all preceding and succeeding operations required to complete the part or product. If sales and losses could be predicted with certainty the problem would be trivial. However, uncertainty exists and some risk must be assumed due to forecast errors.

The Estimate of Future Sales Requirements

No completely satisfactory method of forecasting future sales requirements has been developed, although exponential smoothing forecasting techniques as developed by Brown (35) and Winters (36) appear to be recognized as superior to earlier techniques for the short range problem. Without considering exponential forecasting Scheele, Westerman, and Wimmert (37, p. 41) classify forecasting techniques as:

1. Historical estimate--presuming that what has happened in the past is a good indicator of what will continue to happen in the future.

2. Sales force estimate--presuming that the people closest to the market can best determine its future behavior.

3. Trend lines--projecting into the future increases or decreases which have occurred in the past.

4. Market surveys--sampling to determine the potential sales for a product for which there is no history on which to base a forecast.

5. Correlation--deriving the forecast for a particular activity by relating the activity to a given industry or industries, or to the economy as a whole.

6. Prior knowledge--the substitution of information provided by higher management levels in order to forecast at the lower level.

Exponential smoothing forecasting techniques can be classified under historical estimates since the projected requirements for the next period are dependent upon historical data from the past. For long range forecasting, such as required for new manufacturing facilities planning, exponential smoothing techniques are less effective than for short range forecasting and therefore is not incorporated in the machine requirements model. Rather the model will be developed independent of the sales forecasting technique used by the individual firm.

In long range sales forecasting for a new manufacturing facility two basic situations exist: (1) the facility is to be used for production of an existing product(s); or (2) the facility is to be used for production of a new product(s). In the first case firms may use any of the six types of forecasting except market surveys while for the second case sales force forecasts, market surveys, prior knowledge, or combinations of these methods may be used. Since standardization of the long range sales forecasting method is illogical to assume, the general technique for equipment or machine estimation should be independent of the method used for sales forecast. Rather it will be assumed that the sales forecast is based upon best available data and most logical forecast method for the firm concerned. The decision as to forecasting method may be based upon quantitative analysis or management judgment. In either case in a real world situation it is unrealistic to assume the facilities planner has complete knowledge of the market.

For the case when the new facility is to be used for a new product there is no reliable method for estimating the variance or form of the distribution if the forecast quantity is assumed as a random variable. Separate estimates may be established by more than one technique, by separate groups, or by a number of individuals. Estimation of variance and frequency distribution of sales based upon the distribution of alternative estimates as a sample might be used but there would be no assurance that the true distribution of sales would be similar to that of sales estimates. Therefore such a technique is discarded.

Each independent estimate of sales is the expected quantity of sales by the selected individual study. From decision theory, by the principle of expectation,

$$E(sa) = \sum_{i=1}^{n} P_i(sa_i) \tag{5.1}$$

where,

$E(sa)$ is the resultant expected sales quantity.

(sa_i) is the predicted sales quantity by estimate i.

P_i is the probability of (sa_i), $\sum_{i=1}^{n} P_i = 1$.

n is the number of individual estimates, or sample size.

If $P_1 = P_2 = \ldots = P_n$, E(sa) is the arithmetic average of the individual estimates. If all P_i's are not equal E(sa) is a weighted average. The method of assigning P_i's does not enter into the model, although in practice subjective assignment by management is likely.

E(sa) can also be determined for the case when the new facility is to be used for production of an existing product. Since E(sa) can be established for either new or existing products, E(sa), regardless of the method of determination, will be used to estimate total production requirements for the final operation in the process for production of a product.

The Estimate of Total Production Requirements for an Operation with E(sa) Assumed Known

Let us now consider any inspection operation in the manufacturing process for a single item. By the inspection only three primary decisions are available. A part is good, bad, or reworkable. If reworkable then after rework it is reinspected and the decision process continued until the decision reduces to good or bad. This permits the consideration of each item inspected as being drawn from a universe having a fraction defective p'. The fraction defective from any inspection operation is represented by p.

$$p = \frac{d}{n} = \frac{\text{number of defective items}}{\text{number of items inspected}} . \quad (38, \text{p. } 393)$$

If we now assume the universe from which a sample of size n, the number inspected, is drawn is infinite or if the universe is finite it is presumed that sampling with replacement occurs, then we can represent the distribution of defectives by the binomial distribution. (28, p. 83)

$$P(\frac{d}{n}) = C^n_d (p')^d (1 - p')^{n-d} \tag{5.2}$$

where C^n_d is the number of combinations of n things taken d at a time, d is the number of defectives in the sample, p' is the fraction defective of the universe and (1 - p') is the fraction of good items in the universe. The binomial is a discrete distribution but the binomial distribution can be approximated by the normal distribution whose mean and standard deviation are the same as those for the binomial. This results since $\frac{d}{n}$ is an average and therefore the central limit theorem can be invoked implying that $\frac{d}{n}$ is approximately normally distributed. (38, p. 392) If n is greater than 50 the normal approximation gives good results for .10 - p' $\leq$ .90 and still does fairly well for .10 > p' > .90 so long as np' $\geq$ 5 (28 pp. 87-89).

The mean of the approximating normal distribution, the expected value of the fraction defective, E(p), is p' and the standard deviation, σ_p, is $\sqrt{p'(1-p')/n}$ (38, p. 392). As long as the sample is representative of the universe, E(p) is independent of sample size. However, σ_p is dependent upon and therefore a function of sample size even though p' remains constant.

We now let n, the sample size, represent the number of times the operation under consideration is repeated per year. The production process for a product is a series of individual operations performed in some assigned sequence. Each of these operations may produce defectives which are removed before the next operation commences. Therefore n may vary for all operations other than the first even though the number of items started through the process per year remains constant. We have noted above, however, that p, the fraction defective of an operation, is a random variable. Therefore, since n is equal to the quantity started through the process less all defectives resulting from preceding operations which have been removed, n must also be a random variable for all but the first operation of each process.

We now assume that all defectives found are removed immediately prior to the operation of interest, without loss of generality. This is possible since if for all inspections prior to the operation of interest the defectives removed are accumulated, this accumulated quantity is the total defective for Q, the quantity started through the process. The number of items expected to be passed through the operation of interest would then be Q minus the expected cumulative defectives removed by inspection. Symbolically,

$$E(n) = Q - p_p{}' Q$$

where,

E(n) is the expected number of items passed through the operation of interest per year.

Q is the quantity started through the process per year.

$p_p{}'$ is the fraction defective generated per year by all operations prior to the operation of interest.

One further assumption is now made. If $E(n)p' \leq 5$ or if fewer than 100 good units of a product are required per year it is assumed that this product will be grouped with other low quantity items to establish a product class. This product class will then be used for facility planning purposes rather than the individual products.

Note that as $E(n) \rightarrow 0$, and/or $p' \rightarrow 0.5$, σ_p increases. But by the above we have assumed that the good quantity product after the final operation cannot be less than 100; therefore $E(n) \geq 100$. It follows also that $Q - p_p' Q - p'E(n)$ cannot be less than 100. If we now let $E(n_g)$ represent the expected good items remaining after passing $E(n)$ items through an operation, then $E(n_g) = Q - p_p'Q - E(n)p' = E(n) - E(n)p'$, and by the above assumption $E(n_g) \geq 100$. Furthermore $E(n) = 100$ only when $E(n_g) = 100$ and $p' = 0.0$. Therefore $E(n) - E(n)p' \geq 100$, and $E(n) = 100$ is the lower limit of the quantity expected to pass through the operation of interest.

It is well-known that the normal distribution is symmetrical about its mean. Furthermore, the term $p'(1-p')$ is symmetrical about the value $p' = 0.5$, as illustrated in Figure 9. It follows that if the sample size, n, remains constant the term $p'(1-p')/n$ is symmetrical and in turn $\sqrt{p'(1-p')/n}$. Therefore, if the sample size remains constant, σ_p and $\sigma_{(1-p)}$ are equal, since $\sqrt{p'(1-p')/n} = \sigma_p = \sigma_{(1-p)}$. It has been shown previously that σ_p increases as $n \rightarrow 0$. It follows also that $\sigma_{(1-p)}$ increases as $n \rightarrow 0$.

The objective in a manufacturing operation is to produce $E(n_g)$ good items by passing $E(n)$ items through the operation. The difference, $E(n) - E(n_g)$, is equal to $E(n)p'$, and $E(n_g) = E(n) - E(n)p' = E(n)(1-p')$.

If $E(n_g)$ remains constant and p' is varied E(n) increases as p' increases. It was shown above that for a constant sample size $\sigma_p = \sigma_{(1-p)}$. However, if the objective is to provide $E(n_g)$ good product, E(n) for $p' = (1-p_i)$ is greater than E(n) for $p' = p_i$, for $p_i < 0.50$. But since σ_p increases as $n \to 0$, σ_p when $p' = p_i$ will be greater than σ_p when $p = (1-p_i)$ if $E(n_g)$ remains constant and n is taken as E(n) which satisfies $E(n_g) = E(n)(1-p')$. Therefore for any $E(n_g)$ maximum σ_p must occur for $p' < 0.50$, which implies that maximum variation in the number of items passed through the operation expressed as a fraction of the started quantity will occur for $p' \leq 0.50$.

If p' represents the true fraction defective generated by the operation of interest after installation in the new facility then the required number to pass through the operation to obtain $E(n_g)$ will be $E(n_g)/(1-p')$ which is equal to E(n).

We now investigate the effect of the assumption of a minimum $E(n_g)$ of 100 and the fact that the maximum value of σ_p occurs for $p' < 0.50$. Not only does this permit determination of confidence limits of p for any value of $p' < 0.50$, but also the number of units which the maximum deviation of p within confidence limits represents for $E(n_g) = 100$. This in turn permits determination of the fraction of E(n) represented by this maximum deviation of p. Since σ_p is maximum when $E(n_g) = 100$ and $p' < 0.50$ the maximum deviation expressed as a fraction of E(n) will also be maximum. It remains necessary to determine the specific value of p' at which maximum fraction deviation of E(n) occurs and the value of this fraction deviation. To accomplish this the following procedure is followed.

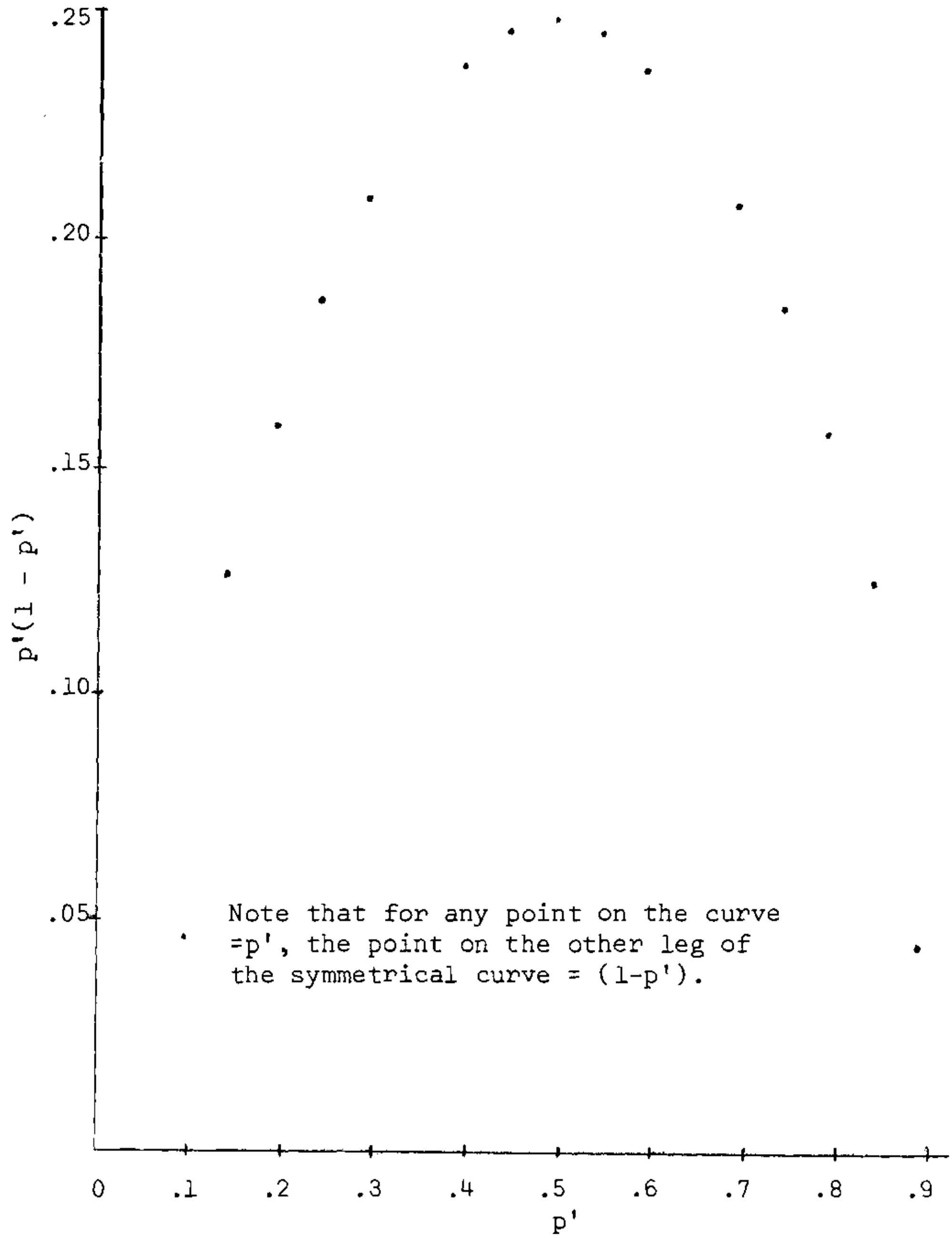

Figure 9. Symmetry of $p'(1 - p')$ About $p' = 0.5$.

$$E(n) = \frac{E(n_g)}{1-p'} = \frac{100}{1-p'}, \tag{5.3}$$

$$\sigma_p = \sqrt{\frac{p'(1-p')}{E(n)}} \tag{5.4}$$

Confidence limits = $p' \pm k\sigma_p$, where

k = The number of standard deviations from the mean of a normal distribution corresponding to the desired confidence limit.

Maximum deviation as a fraction of E(n) =

$$E(n)k\sigma_p/E(n) = k\sigma_p \tag{5.5}$$

Table 1 presents tabulated results for k = 1.96, equal to 95 per cent confidence limits. Under these conditions, maximum deviation expressed as a fraction of E(n) is approximately 0.0755, at a p' of approximately 0.35, by observation. As E(n) increases to more than 100, and/or p' deviates from $\approx$ 0.35 the error is reduced. For example at $E(n_g)$ = 100 and p' = 0.0476 maximum expected deviation expected at 95 per cent confidence is approximately 0.042 E(n). If $E(n_g)$ is 1000 and p' = 0.0476, maximum expected deviation is reduced to approximately 0.013 E(n).

We can conclude from the above that if $E(n_g)$ is known and p' estimated by the best possible means the maximum deviation we can expect in n from E(n) with 95 per cent confidence is approximately .0755 E(n), if $E(n_g) \geq 100$ and $p' \leq 0.5$. As $E(n_g) \gg 100$ and if p' deviates from 0.35 the maximum expected deviation is reduced. The more likely values to be encountered in practice are $E(n_g) >> 100$ and $p' << .35$, and therefore it is reasonable to expect the deviation to be less than 0.0755 E(n).

Table 1. Maximum Deviation Within 95 Per Cent Confidence Limits for $E(n_g) = 100$ and Various Values of $p' \leq 0.50$

p'	E(n)	σ_p	Maximum Deviation of p Within 95 Per Cent Confidence Limits = 1.96 σ_p
.5	200	.0354	.0694
.398	166	.0380	.0745
.375	160	.0382	.0749
.355	155	.0385	.0755
.333	150	.0384	.0753
.300	143	.0383	.0751
.248	133	.0374	.0733
.200	125	.0358	.0702
.1667	120	.0340	.0666
.0476	105	.0215	.0421

It is now necessary to establish values which can be used as estimators for the above parameters. Since it has been assumed earlier that E(sa) will be used as the value of required sales, this must also be the value of $E(n_g)$ for the final operation or finished product. For any operation other than the last $E(n_g)$ will be equal to $u_j E(sa)$, where

u_j is the number of part j's in the finished product, plus the expected cumulative defectives for all operations following the operation of interest. It was shown above that the expected value of fraction defective for any operation is equal to p' for that operation. $E(n_g)$ for any operation of interest is therefore equal to the sum of $u_j E(sa)$ and the sum of all defectives equivalent to p' for each operation following the operation of interest. Symbolically,

$$E(n_g)_i = u_j E(sa) + \sum_{i=1}^{n} E(n)_i p'_i \;, \tag{5.6}$$

and by substitution into Equation 5.3

$$E(n)_i = \frac{E(n_g)_i}{1-p_i'} \;, \tag{5.7}$$

The maximum likelihood estimator of the parameter p' for the approximating normal distribution from sample data is $\bar{p}$, the average sample defective (34, p. 89). It must be kept in mind, however, that for the problem under consideration it may not be possible to secure representative sample data since process installation has not been accomplished. The more general $\hat{p}$ will therefore be used and will be assumed to be established by the best available method, preferably as $\bar{p}$ from sample data.

Since $E(n_g)$ and $E(n)$ are dependent upon p's and an estimated value must be used for each p', $E(n_g)$ and $E(n)$ must also be estimates which will be designated $\hat{n}_g$ and $\hat{n}$. Furthermore, since estimates must be used for parameters for calculating σ_p, only an estimate $\hat{\sigma}_p$ can result, and

$$\hat{\sigma}_p = \sqrt{\frac{\hat{p}(1-\hat{p})}{\hat{n}}} \quad .$$

If the estimate of machines required to produce n_g based upon $\hat{p}$ is in error by the maximum expected deviation of n equal to $k\sigma_p\hat{n}$, and the calculated machine requirement is a whole number, overtime equivalent to .0755• (normal working hours) would be required to overcome the effect of the error. However, as noted in the literature search section of Chapter II, some reserve above normal requirements is generally present due to the calculated machine requirements being a mixed number which most often results in a decision to provide the next higher whole number of machines. Therefore, it is likely that all or a portion of additional machine time resulting from deviation of n is available without increasing machine or overtime investment. Therefore, it is concluded that $\hat{n}$ provides a satisfactory value of n to be used in determining N, the number of machines. Furthermore, since $\hat{n} = \hat{n}_g/(1-\hat{p})$, $\hat{n}$ can be determined from $\hat{n}_g$ and $\hat{p}$. As we have concluded earlier p can be either the fraction defective from a single operation or the cumulative fraction defective from a series of operations considering the input to the first operation performed in the series as n. We now consider the series of consecutive operations starting with the operation of interest and ending with the final operation of the process. For this series of operations $E(sa) = \hat{n}_g$ and $E(sa)/(1-\hat{p}) = \hat{n}_i$ is the estimate of n_i, the input to operation i, the operation of interest.

It remains necessary to determine $\hat{p}$, the estimate of p'. The maximum likelihood estimator of p has been shown to be the sample average, $\bar{p}$,

$$\bar{p} = \frac{\text{defectives in the total sample}}{\text{number of items in the total sample}}$$

In general, let $\hat{p}_i = \hat{c}_i/\hat{n}_i$, where $\hat{c}_i$ is the estimated number of defectives resulting from operation i only and $\hat{n}_i$ as before is the estimated number of items entering operation i, from which $\hat{c}_i$ defectives are estimated to be produced. Then $\hat{n}_i = \hat{c}_i + \hat{c}_{i+1} + \ldots + \hat{c}_{f-1} + \hat{c}_f + u_jE(sa)$, where $\hat{c}_f$ is the defectives estimated to result from the final operation f, after which $u_jE(sa)$ good items are desired, where u_j is the number of part j's in the final product. Then the estimated cumulative defective $\hat{p} = (\sum_{x=1}^{f} c_x)/\hat{n}_i$, and $\hat{p}_i = \hat{c}_i/\hat{n}_i$. Furthermore, $u_jE(sa) = \hat{n}_i - \sum_{x=1}^{f} c_x = (1-\hat{p})\hat{n}_i$. Solving for $\hat{n}_i$,

$$n_i = \frac{u_jE(sa)}{1-\hat{p}}, \tag{5.8}$$

which agrees with the earlier estimate of n_i.

Now for i = 1, the first operation in the process, n_i is the estimated number of items to start through the process to result in E(sa) good items from the final operation of the process identical to $\hat{n}$ above when $\hat{p}$ is the estimated fraction defective from the total process having $\hat{n} = Q$, the total input to the process.

We now derive a distribution for n_i, the input to operation i, resulting from $Q = u_j E(sa)/(1-\hat{p})$ and p_p the fraction defectives resulting from process operations prior to operation i. As shown above $E(n) = Q - p_p'Q$, where $E(n)$ is the expected number of items passed through the operation of interest, now identified as operation i. Furtheremore $E(n)$ is estimated by $\hat{n}_i$ and p_p' by $\hat{p}_p$. Therefore,

$$\hat{n}_i = Q\ (1 - \hat{p}_p). \tag{5.9}$$

But Q is a constant and $1-p_p$ a random variable distributed normally with mean $(1 - \hat{p}_p)$ and standard deviation,

$$\sigma_{(1-p_p)} \doteq \sqrt{\frac{\hat{p}_p(1-\hat{p}_p)}{Q}}\ . \tag{5.10}$$

$Q(1-p_p)$ being a constant times a random variable must have the same distribution as the random variable, in this case normal, with mean equal to the constant times the mean of the random variable and standard deviation equal to the constant times the standard deviation of the distribution of the random variable. Using the estimates of the mean and standard deviation of the binomial distribution for p_p,

$$\hat{\mu}_{Q(1-p_p)} = Q(1-\hat{p}_p) \tag{5.11}$$

$$\hat{\sigma}_{Q(1-p_p)} = Q\sqrt{\frac{\hat{p}_p(1-\hat{p}_p)}{Q}} \tag{5.12}$$

$$= \sqrt{Q\hat{p}_p(1-\hat{p}_p)} \ .$$

But $Q(1-p_p) = n_i$ and therefore n_i is approximately distributed normally with

$$\mu_{n_i} \doteq Q(1-\hat{p}_p) = \hat{n}_i \tag{5.13}$$

and

$$\sigma^2_{n_i} \doteq Q\hat{p}_p(1-\hat{p}_p) \ . \tag{5.14}$$

n_i, the number of items passing through the operation of interest, is equal to P, the operation cycles per time period, for that operation and therefore the distribution of n_i represents the distribution of P, the total production requirements, or operation cycles per time period.

CHAPTER VI

DETERMINATION OF THE DISTRIBUTION OF THE USE FACTOR

The distributional nature of the use factor must be defined before it is discussed. There is a tendency in the literature to confuse this factor with a utilization factor. A relationship exists but the two factors are not the same. Rather, the use factor tends to be the upper limit of the utilization factor. The utilization factor is defined as the ratio of hours of equipment use to total hours available over some unit time period, frequently one production year. Utilization is therefore dependent upon the load or demand requirements against the machine, as well as the ability of the machine to perform without malfunction or stoppage for preventive maintenance. If load requirements were sufficient to place a demand on the machine at all times when it was available, the utilization factor would approach a maximum ≤ 1. At the maximum level the ratio for utilization could be rewritten as the ratio of the hours of equipment use to the sum of hours of equipment use plus hours lost for maintenance, since the denominator in the maximum case would be equal to total production hours available over the unit time period. Since most applications of the utilization factor in planning do not consider the distributional nature of the utilization factor, but rather the average or expected value, the maximizing ratio could be rewritten as the average maximum expected hours of equipment use to average maximum hours of equipment use plus average hours lost for maintenance. The sum of the denominator continuing to be equal to expected total production hours available over the unit time period.

Now note that if we write the ratio for the unit time period as

$$\frac{\text{Expected hours of equipment use}}{\text{Expected hours loss to maintenance + Expected hours of equipment use}}$$

the value of the ratio is the same as the upper bound of the expected utilization factor. Note further that this ratio is not dependent upon demand for machine use nor upon production hours per unit time period except that production hours per unit time period is the limit of the denominator. If the denominator exceeds the production hours per unit time period, additional equipment must be provided or the extra demand must be accomplished during overtime. The ratio, however, remains fixed over all demand levels. Because this ratio is independent of demand, it is much more meaningful in equipment planning when it is desired to sum the demand requirements against a machine model to determine the number of machines required, which is the problem under consideration in this study. To distinguish this ratio we designate it the average use factor. If we now eliminate the use of averages in the ratio, we obtain

$$\frac{\text{Hours of equipment use}}{\text{Hours lost to maintenance + Hours of equipment use}} = C, \text{ the use factor.}$$

C is a continuous random variable having limits 0 and 1. Hours of equipment use is the sum of the hours during the unit time period that the equipment is assigned and is measured in standard time units. Note that by this definition time consumed due to operating or machine delays corresponding to allowances in the standard time are considered a part of hours of equipment use.

If we invert the ratio, and designate it U, it remains a random variable but with limits 1 to ∞. For hours lost to maintenance to be zero, i.e., U = 1, the reliability of the machine would need to be perfect, which is unlikely.

Now note that in the initial model the value of interest is 1/C. If we substitute U for this, we can now write the model,

$$N = \frac{1}{H_a} f(t_a, P, U) \tag{6.1}$$

N thus being a function of t_a, P, and U.

Additional factors should be noted relative to hours lost to maintenance. In setting a time standard an allowance is normally made for machine delay. This frequently allows for minor stoppages for repair or adjustment of the machine. In practice, some firms continue to pay against standard hours until the machine down time for maintenance exceeds some fixed limit, often 15 to 30 minutes. After the predetermined down time limit has been reached on an individual stoppage, the operator is placed on straight time. If such a practice is followed by the firm under consideration, hours lost to maintenance should not include the initial time equal to the allowed limits for any instance of maintenance. This is because this initial time is already allowed for in the standard time and as time against standard which is an element in the determination of the average time variable in the model.

A second practice which may affect hours lost to maintenance is that of having a portion of the maintenance crew scheduled to work at hours during which production is not scheduled. Certain normal or scheduled maintenance may be planned for this non-production time. As a result, the maintenance accomplished does not interfere with use during production time. Since interference with production time is not present, maintenance time expended during the period should not be included as time lost to maintenance. However, if overtime is required for the maintenance crews, such overtime should be charged as time lost to maintenance, since the reason for overtime is to avoid reduction of effective production time beyond that saved by planned non-overtime maintenance scheduling.

Since it is desirable to maximize C, when its inverse U is the variable of interest it is desirable to seek minimization of U. Under ideal conditions $U = 1.0$. Since ideal conditions are virtually never realized in practice, $U > 1.0$, but the objective of management is to have $U \to 1.0$, within the bounds of optimizing economies.

The Distribution of U

As defined above, U is largely dependent upon reliability characteristics of the equipment, replacement policy of the firm, and maintenance practices. Closely related are the factors of set-up time and scheduling inefficiencies. These last two, however, result primarily due to multi-order or multi-product effects and will be considered later when analyzing the combination of individual operation requirements to determine total machine model requirements.

Reliability is defined as "the probability that a device will perform, without failure, a specified function under given conditions for a specified period of time." (39,p. 21.) Three types of failure are normally considered: (1) wearout failure, a progressive reduction of capability until failure; (2) chance failure, those failures which are unpredictable and equally likely to occur at any time in life; and (3) initial failure or infant mortality, the failures due to "newness" which normally are corrected during run-in (39, pp. 21, 22 and 40, pp. 134, 135). Initial failures will not be considered as affecting operating conditions since they are eliminated during "shake-down," or running in before placing the equipment on-line for normal production. A number of distributions have been found applicable to chance failures, the choice being dependent upon the time-to-failure data recorded against, or estimated for, the component These include the exponential, Weibull, gamma, and extreme value distributions (40, p. 135ff.). Wear out failures most frequently tend to follow a normal probability distribution about a mean life time (39, p. 23). By the "product rule," the probability of the successful operation of a system (the machine) is equal to the product of the probabilities of successful operation of each independent and mutually exclusive component, or sub-system, i.e., the product of the reliabilities of the components and sub-systems. If redundant units are incorporated to reduce the probability of system failure reliabilities incorporating sub-system redundancy effect are used in applying the product rule. Component interaction also affects reliability. Interaction may be environmental, such as vibration causing electronic failure, or functional when a degree of non-compatibility exists between components (40, pp. 5 and 220ff.). It is theoretically possible to determine the machine (system) reliability by combining the reliability distributions of all components. In practice this is not often practical for most production equipment due to lack of data for components and the expense involved in developing and analyzing data. However, reliability of certain major, or critical, components or sub-systems may be determined, particularly for wear-out failures and replacement of these components or sub-systems incorporated in the preventive maintenance program on a time interval or number of operating cycles interval basis (41).

Reliability is concerned with the frequency of system stoppage or malfunction. When malfunction occurs in a unit of production equipment, maintenance is required. Availability of replacement parts, of repair facilities, and maintenance personnel at time of stoppage will each affect "downtime." Actual repair time for a malfunction maintenance operation will vary in the same manner as discussed for production operation time in Chapter IV and as a result can be approximated by a normal distribution. Actual times for periodically scheduled preventive maintenance operations will also be distributed approximately as a normal distribution in the same manner as for malfunction repair times. Periodically scheduled preventive maintenance operations are normally scheduled to be performed after the passage of a fixed period of time following the previous maintenance operation occurrence, frequently expressed in number of working hours or working days, degree of use during this period not being considered. If this schedule is maintained, deviation of time between identical maintenance operations on a machine is inconsequential and as a result maintenance time per unit time period is distributed as the sum of actual times for individual occurrences which is the sum of a series of normal distributions. In practice, however, an assignment of operations is made to a preventive maintenance man or crew on a fixed sequence basis. The total period assigned to the cycle being equal to the sum of average time for each operation. From the above, it is known that individual operation actual times are random variables normally distributed about the average and therefore the time between occurrences of the operation assigned to a fixed position in the cycle must be the sum of a number of independent normally distributed random variables. But the sum of a number of independent normally distributed random variables is also normally distributed (45, p. 94) and therefore time between occurrences of a periodically scheduled maintenance operation must also be normally distributed. The preventive maintenance time per unit time period for a single operation must equal the product of number of occurrences and time per occurrence. But both of these factors from the above are normally distributed random variables and independent in the statistical sense. The product of two independent normally distributed random variables is approximately normally distributed (28, p. 74). For chance failures, Lloyd and Lipow state as a general rule that the estimate of the reliability function R, defined as the probability of success, "is approximately normally distributed when n, the sample size, is large." (40, p. 192.) But R = 1 - F where F is the failure time distribution function, and if R is normally distributed, F will also be normally distributed. Again, time per unit time period for chance failure maintenance must equal the product of the number of occurrences and time per occurrence and since each is an independent normally distributed random variable, the product is approximately normally distributed.

The major class of maintenance occurrence remaining is that associated with preventive maintenance scheduled based upon the number of recurring production cycles since the previous maintenance. This is dependent upon demand and if we assume demand constant, time between occurrences is assumed to be normally distributed with mean equal to the product of the number of production cycles and the mean time per production cycle, and standard deviation equal to the product of the number of production cycles and the standard deviation of the individual production cycle time (by the additivity rule for variances). If on the other hand, demand is approximately normally distributed as indicated in Chapter V, the distribution of time between maintenance operations will also be approximately normal since it is the product of two independent normal variables, demand and time per operation cycle (28, p. 74).

If time required per unit time period for each of the three types of maintenance--periodically scheduled preventive, cyclic scheduled preventive, and chance occurrence--is considered independent and if each can be approximated by a normal distribution, then the total time for maintenance per unit time period can be approximated by a normal distribution by the rule that the sum of independent normally distributed random variables is also normally distributed, as stated above. The total time for maintenance per unit time period is the time lost for maintenance in our expression for U.

Hours of equipment use is going to be a random variable resulting from the additive influences of a number of independent and dependent variables including those associated with equipment reliability, firm maintenance policies, and replacement policies as well as demand for production and average time per production operation. An extension of the central limit theorem says that "if x is the sum of a number of independent variables $y_1, y_2, \ldots, y_n$, then whatever the distributions of the individual y's (subject to very general restrictions), the distribution of x approaches the normal form as the number of variables n gets larger and larger." (42, pp. 230, 231.) Feller states, "the central limit theorem holds also for large classes of dependent variables." (43, p. 240.) By this extension we can expect the hours of equipment use to be distributed approximately normally, particularly if the period production demand remains reasonably stable over the measured time interval which is inherent in our earlier assumption of E(sa) as annual demand. This conclusion is reinforced by the fact that the probability of equipment use plus the probability of down time for maintenance must sum to one, or P (use time) = 1 - P (down time). Since P (down time) was shown above to be normally distributed, then P (use time) must also be normally distributed. Also, t_a per operation cycle and n_i operation i input per unit time period were shown to be approximately normally distributed. By reference above, the product $n_i t_a$ is approximately normally distributed and since $n_i t_a$ equals use time per unit time period, this must also be approximately normally distributed.

Not let H_m be the hours lost to maintenance and let H_u be the hours of equipment use. Then,

$$U = \frac{H_m + H_u}{H_u} = \frac{H_m}{H_u} + 1 , \qquad (6.2)$$

But H_m, except for time required for that preventive maintenance scheduled on the basis of fixed time intervals, will have a dependency relationship with H_u, related to operating cycles or chance associated with equipment operation. This dependency may be approximated by a linear regression equation, $H_m = c + mH_u$, where c is the constant or intercept value of H_m and m is the slope of the regression line, representing the rate of change in H_m as H_u increases. Substituting this into the above equation,

$$U = \frac{c + mH_u}{H_u} + 1 \qquad (6.3)$$

$$U = m + 1 + c[\frac{1}{H_u}] \quad \begin{array}{l} 1 \leq U < \infty \\ 0 \leq H_u \leq \infty \end{array} \qquad (6.3)$$

The linear regression curve of H_m on H_u can be estimated from sample data using the method of least squares. This assumes that the "best" system of values for the coefficient m is one which renders the sum of squares of the deviations of sample values of H_m from the equation estimated value of H_m for all values of H_u, minimum (52, p. 116).

It was shown above that H_m and H_u are each approximately normally distributed. The joint frequency distribution is bivariate normal, and substituting into the general expression of Burington and May (52, p. 127),

$$f(H_u, H_m) = \frac{1}{2\pi\sigma_{H_u}\sigma_{H_m}\sqrt{1-r^2}} \exp(-G/2) \tag{6.4}$$

where,

$$G = \frac{1}{1 - r^2}\left[\frac{(H_u - \bar{H}_u^2)}{\sigma H_u^2} - \frac{2r(H_u - \bar{H}_u)(H_m - \bar{H}_m)}{\sigma H_u \; \sigma H_m} + \frac{(H_m - \bar{H}_m)^2}{\sigma H_m^{\;2}}\right]$$

and r is the correlation coefficient of H_u and H_m.

$$r = \mathrm{cov}(H_u, H_m)/\sigma_{H_u}\sigma_{H_m} \tag{6.5}$$

where,

$\mathrm{cov}(H_u, H_m)$ = Covariance between the H_u's and H_m's,

$$\doteq \frac{\sum(H_{u_i} - \bar{H}_u)(H_{m_i} - \bar{H}_m)}{k} = \frac{[\sum(H_{u_i} H_{m_i}) - k\bar{H}_u\bar{H}_m]}{k}, \tag{6.6}$$

and k is the sample size (52, p. 118). The marginal frequency function for H_u is

$$f_1(H_u) = \frac{1}{\sigma_{H_u}\sqrt{2\pi}} \exp[-\frac{(H_u - \bar{H}_u)^2}{2\sigma_{H_u}^2}] . \tag{6.7}$$

The conditional frequency distribution of H_m, relative to a fixed value of H_u is,

$$f(H_m|H_u) = \frac{f(H_u, H_m)}{f_1(H_u)}$$

$$= \frac{1}{\sigma_{H_m}\sqrt{2\pi(1-r^2)}} \exp[\frac{1}{2\sigma_{H_m}^2(1-r^2)} (H_m - m_2(H_u))^2] \tag{6.8}$$

where,

$$m_2(H_u) = \bar{H}_m + (r\sigma_{H_m}/\sigma_{H_u})(H_u - \bar{H}_u).$$

$f(H_m|H_u)$ then is a normal frequency function in H_m with mean $m_2(H_u)$ and standard deviation $\sigma_{H_m}\sqrt{1-r^2}$. Since H_m and H_u are each normal, the regression of H_m on H_u is linear, and the conditional variance of H_m is independent of the value assumed by H_u (52, p. 127).

For subjective estimation purposes when the linear regression equation cannot be determined from sample data, c can be taken as the estimated time required for fixed time interval scheduled preventive maintenance based upon frequency and estimated average time for occurrence, and m is estimated as the rate of change of H_m to H_u based upon best available information.

CHAPTER VII

COMBINATION OF THE RANDOM VARIABLES $\bar{t}_a$ AND P

The generalized form of the initial model for determining machine requirements for a single operation has been shown by Equation 6.1 to be

$$N = \frac{1}{H_a} f(\bar{t}_a, P, U)$$

where $\bar{t}_a$ and P are each approximately normally distributed. For the present, the distribution of U is undefined although some available empirical data can be offered that indicates the assumption of normality may be justified for some classes of processes (see Tables 2 and 3). It also remains necessary to determine a means of summarizing individual operation requirements to determine total requirements for a specific machine model.

It is possible to consider the joint distribution of the three random variables as a trivariate distribution (if U is normal as a trivariate normal) and solve for machine requirements for each operation under a selected decision rule. Individual operation requirements can then be summed as an estimate of total requirements.

Another alternative is to combine each of the random variables individually across all operations on the machine model. Two problems arise in this alternative. First, the random variable U is primarily

Table 2. Monthly Use Values for Georgia Institute of Technology Burroughs 220 Computer

Month	Production Time-Hours H_u	Maintenance Time—Hours H_m	Total Machine Hours $H_m + H_u$	U
1960 August	153.26	126.86	279.12	1.821
September	157.40	122.95	280.35	1.781
October	159.18	118.77	277.95	1.746
November	155.18	124.33	279.51	1.801
December	162.12	119.09	281.21	1.735-
1961 January	219.42	112.65	332.07	1.513
February	169.39	73.75	243.14	1.435
March	133.15	108.20	242.35	1.820
April	166.32	102.02	268.40	1.614-
May	222.11	86.86	308.97	1.319
June	162.46	132.07	294.53	1.813-
July	174.26	85.33	259.59	1.490-
August	312.25	115.12	427.37	1.369-
September	247.11	87.04	334.15	1.352
October	202.55	134.98	337.53	1.666
November	231.17	113.12	344.29	1.489
December	261.21	143.12	414.33	1.586
1962 January	215.05	101.98	317.03	1.474
February	274.30	218.06	492.36	1.795-
March	366.32	148.20	514.52	1.405-
April	393.10	103.96	497.06	1.264
May	447.48	102.58	550.06	1.229
June	331.12	141.99	437.11	1.429-
July	299.50	152.52	452.02	1.509
August	387.25	165.84	553.09	1.428
September	323.20	93.29	416.49	1.289-
October	333.35	178.23	511.58	1.535-
November	344.14	134.18	478.32	1.390-
December	245.43	122.98	368.41	1.501
1963 January	190.36	275.92	466.28	2.449
February	349.33	138.18	487.51	1.396-

Summary: $\bar{x} = 1.5313$.

$\hat{\sigma} = 0.382$.

X^2 for 11 degrees of freedom at 0.01 fiducial point = 24.73. Therefore, data does not deviate significantly from a normal distribution.

Table 3. Monthly Use Values for Remington Rand 1101

Month		Production Time-Hours H_u	Maintenance Time—Hours H_m	Total Machine Hours $H_m + H_u$	U
1960	August	133.50	73.35	206.85	1.549
	September	134.44	80.59	215.03	1.599
	October	164.50	73.98	238.48	1.450-
	November	67.10	45.17	112.27	1.822
	December	27.43	38.53	65.96	2.405-
1961	January	65.53	19.75	85.28	1.301
	February	133.36	40.67	174.03	1.305
	March	111.30	62.79	174.09	1.564
	April	128.02	31.19	159.21	1.244
	May	107.31	72.83	180.14	1.679
	June	130.40	83.00	213.40	1.637
	July	109.28	56.90	166.18	1.521
	August	155.26	46.67	201.93	1.301
	September	106.35	69.85	176.20	1.657
	October	153.02	50.63	203.65	1.331
	November	111.04	77.82	188.86	1.701
	December	76.01	89.04	165.05	2.171
1962	January	130.57	83.44	214.01	1.639
	February	59.21	98.06	157.27	2.656
	March	94.11	94.99	189.10	2.009
	April	130.34	56.59	186.93	1.434
	May	146.30	50.35	196.65	1.344
	June	121.47	66.13	181.60	1.544
	July	149.05	44.10	193.15	1.296
	August	125.44	76.36	201.80	1.609
	September	109.51	60.89	170.40	1.556
	October	127.41	83.73	211.14	1.657
	November	147.15	33.50	180.65	1.228
	December	122.00	25.12	147.12	1.206
1963	January	120.55	71.15	191.70	1.590
	February	123.00	54.00	177.25	1.441

Summary: $\bar{x} = 1.595.$

$\hat{\sigma} = 0.322.$

X^2 for 11 degrees of freedom at 0.01 fiducial point = 24.73. Therefore, data does not deviate significantly from a normal distribution.

dependent upon the machine model, while the random variables $\bar{t}_a$ and P are primarily dependent upon the operation and upon the sales forecast and product process, respectively. Furthermore, so long as the materials, operators, and general complexity of operations performed remain approximately constant U will approach a constant value for all operations. If U is a constant the joint distribution function will reduce to

$$N = \frac{U}{H_a} f(\bar{t}_a, P), \tag{7.1}$$

and the trivariate probability function is reduced to a bivariate.

Now note that for an individual operation ($\bar{t}_a$ x P) equals the average time to complete P operations. If U = 1,

$$N_I = \frac{1}{H_a} f(\bar{t}_a, P), \tag{7.2}$$

where N_I is used to indicate an incomplete determination of N. It is incomplete due to the elimination of the consideration of U. However, if we determine N_I for all operations required on a machine model, we can then multiply the machine model N_I by U, assumed constant, to obtain N for the machine model. This results in a model of the general form

$$N = \frac{1}{H_a}(U)\ f[\sum_{j=1}^{n} (\bar{t}_{a_j}, P_j)] , \tag{7.3}$$

where n is the total number of operations performed upon the machine.

Establishment of an Equivalent U

If the assumption of invariance of U for all operations on the machine model is not realistic for a specific situation, the following procedure may be used to obtain an equivalent relationship. First group operations which can be considered to have equal U due to similarity of material and/or operation complexity. If operator skills assigned are expected to vary between operations the grouping should take this into consideration if it may affect U. Variation in skills between operations is normally assumed not to be significant in practice due to the assignment of similar skills to work of similar complexity on identical machines. Therefore, variation in skills would be considered not to exist in the following discussion.

Now $\bar{t}_a P$ is an estimate of the number of operating hours required for an operation, or equipment use hours, H_u. By our earlier assumption of linear regression approximation of hours lost to maintenance from hours of equipment use U is dependent upon $\bar{t}_a P$, the estimate of H_u (see Equation 6.3). We therefore use $\bar{t}_a P$ as an estimate of the relative weight of each U_z as it affects the machine model U. Letting $\hat{U}$ be the estimate of U, for each machine model,

$$\hat{U} = \sum_{z=1}^{k} \frac{\bar{t}_{az} P_z U_z}{\sum_{z=1}^{k} \bar{t}_{a_z} P_z} \tag{7.4}$$

where k is the total number of groups the operations on the machine model are subdivided into, and

$$\bar{t}_{a_z} P_z = \sum_{j=1}^{h_z} \bar{t}_{a_j} P_j = h_z \overline{(\bar{t}_{a_j} P_j)} \qquad (7.5)$$

where,

j = The individual operation within group z.

h_z = The number of operations within group z.

$\overline{(\bar{t}_{a_j} P_j)}$ = The average value of $\bar{t}_{a_j} P_j$ for the group z,

$$= \sum_{j=1}^{h_z} \frac{\bar{t}_{a_j} P_j}{h_z}$$

By the method of establishment the groups must be mutually exclusive and by the usual work measurement assumption of basic motion elements being independent of preceding or succeeding operations, independent in the statistical sense.

By the rules of linear combination of independent events and the theorems; $E(c) = c$, if c is a constant, $E(X \pm Y) = E(X) \pm E(Y)$ and $E(XY) = E(X)E(Y)$, if X and Y are statistically independent (34, p. 36 and p. 80), we can establish expressions for the parameters of the machine model U based upon group parameters.

$$E(U) = \sum_{z=1}^{k} c_z E(U_z) \qquad (7.6)$$

where,

$$c_z \text{ is a weighting constant } = \frac{E(\bar{t}_{a_z} P_z)}{\sum_{z=1}^{k} E(\bar{t}_{a_z} P_z)} .$$

It follows that,

$$\mu_U = \sum_{z=1}^{k} c_z \mu_{U_z} = \sum_{z=1}^{k} \left\{ \frac{E(\bar{t}_{a_z} P_z)}{\sum_{z=1}^{k} E(t_{a_z} P_z)} \right\} \mu_{U_z}, = E(U) \qquad (7.8)$$

and

$$\sigma_U^2 = \sum_{z=1}^{k} c_z^2 \sigma_{U_z}^2 = \sum_{z=1}^{k} \left(\frac{E(\bar{t}_{a_z} P_z)}{\sum_{z=1}^{k} E(\bar{t}_{a_z} P_z)} \right)^2 \sigma_{U_z}^2 \qquad (7.9)$$

The Distribution of $f(\bar{t}_{a_j} P_j)$

Gardiner (53) considers a function f(x, y, z) of the random variables x, y, and z and a function g(r, s, t) of the random variables r, s, and t, with f(x, y, z) expanded in a Taylor Series about the point (x_0, y_0, z_0) and g(r, s, t) expanded in like manner around the point (r_0, s_0, t_0). The expansion gives

$$f(x, y, z) = f(x_0, y_0, z_0) + [(x-x_0)\frac{d}{dx} + (y-y_0)\frac{d}{dy}$$

$$+ (z-z_0)\frac{d}{dz}]_o f + [(x-x_0)\frac{d}{dx} + (y-y_0)\frac{d}{dy} + (z-z_0)\frac{d}{dz}]_0^2 f/2! + \ . \ . \ .$$

and

$$g(r, s, t) = g(r_0, s_0, t_0) + [(r-r_0)\frac{d}{dr} + (s-s_0)\frac{d}{ds}$$

$$+ (t-t_0)\frac{d}{dt}]_0\ g + [(t-r_0)\frac{d}{dr} + (s-s_0)\frac{d}{ds}$$

$$+ (t-t_0)\frac{d}{dt}]_0^2\ g/2! + \ldots$$

Next, letting $E(X) = x_0$, $E(Y) = y_0$, etc., and supposing that $E[f(x, y, z)] = f(x_0, y_0, z_0)$ and $E[g(r, s, t)] = g(r_0, s_0, t_0)$ to a good approximation Gardiner forms $f - E(f)$ and $g - E(g)$, multiplies these together, drops all terms higher than the second order, and obtains,

$$[f - E(f)][g - E(g)] = (x-x_0)(r-r_0)f'_x g'_r + (y-y_0)(s-s_0)f'_y g'_s$$

$$+ (z-z_0)(t-t_0)f'_z g'_t + (x-x_0)(s-s_0)f'_x g'_s$$

$$+ (x-x_0)(t-t_0)f'_x g'_t + (y-u_0)(r-r_0)f'_y g'_r$$

$$+ (z-z_0)(r-r_0)f'_z g'_r + (z-z_0)(s-s_0)f'_z g'_s \ ,$$

where $f'_x = \frac{df}{dx}$ evaluated at x_0, y_0, z_0, $g'_t = \frac{dg}{dt}$ evaluated at r_0, s_0, t_0, etc. The expectation of this expression is then,

$$\text{cov}(f, g) \doteq f'_x g'_r \text{cov}(x,r) + f'_y g'_s \text{cov}(y,s) + f'_z g'_t \text{cov}(z,t)$$

$$+ f'_x g'_s \text{cov}(x,s) + f'_x g'_t \text{cov}(x,t) + f'_y g'_r \text{cov}(y,r)$$

$$+ f'_y g'_t \text{cov}(y,t) + f'_z g'_r \text{cov}(z,r) + f'_z g'_s \text{cov}(z,s) \ .$$

Suppose now that $f(x, y, z) = f(s_i, y_i, z_i)$ and that $g(r, s, t) = f(x_j, y_j, z_j)$. With an obvious notation we may write

$$\text{Cov}\ (f_i, f_j) \doteq (f'_x)^2\ \text{Cov}\ (x_i, x_j) + (f'_y)^2\ \text{Cov}\ (y_i, y_j) + (f'_z)^2$$

$$\text{Cov}\ (z_i, z_j) + 2\ f'_x f'_y \text{Cov}\ (x_i, y_j) + 2'_x f'_z f\ \text{Cov}\ (x_i, z_j)$$

$$+ 2\ f'_y\ f'_z\ \text{Cov}\ (y_i, z_j)$$

If $i = j$, the equation becomes

$$\text{Var}\ (f) \doteq (f'_x)^2\ \text{Var}\ (x) + (f'_y)^2\ \text{Var}\ (y) + (f'_z)^2\ \text{Var}\ (z)$$

$$+ 2\ f'_x\ f'_y\ \text{Cov}\ (x, y) + 2\ f'_x\ f'_z\ \text{Cov}\ (x, z) + 2\ f'_y\ f'_z\ \text{Cov}\ (y, z)\ .$$

Suppose now that $f = (ax + b)(cy + d)$ where a, b, c, and d are non-random variables or constants and the coefficient of variation for both x and y is small; then the last equation reduces to the special case,

$$Var(f) \doteq [a(cy_0+d)]^2 Var(x) + [c(ax_0+b)]^2 Var(y) + 2ac(cy_0+d)(ax_0+b)cov(x,y).$$

If $a = c = 1$ and $b = d = 0$, $f = xy$ and $Var(f) \approx {y_0}^2 Var(x) + {x_0}^2 Var(y) + 2x_0 y_0 cov(x,y)$, which is the general case for $f(\bar{t}_{a_j} P_j)$.

By earlier assumptions $\bar{t}_{a_j}$ and P_j are independent random variables with small coefficient of variation. For independent random variables the covariance is zero as is the correlation coefficient (54, p. 108). Therefore, substituting $\mu_{\bar{t}_{a_j}}$ for x_0 and μ_{P_j} for y_0 and completing the substitution in a like manner,

$$Var(\bar{t}_{a_j}, P_j) \doteq {\mu_{P_j}}^2 Var(\bar{t}_{a_j}) + {\mu_{\bar{t}_{a_j}}}^2 Var(P_j). \quad (7.10)$$

Furthermore, since as shown earlier for independent random variables $E(X \cdot Y) = E(X)E(Y)$, from which $\mu_{X,Y} = \mu_X \mu_Y$ since $E(X) = \mu_X$ (52, p. 56).

$$\mu_{\bar{t}_{a_j} P_j} = \mu_{\bar{t}_{a_j}} \mu_{P_j} . \quad (7.11)$$

Duncan (28, p. 74) states that if the distribution of individual random variables is normal, the product will be approximately normally distributed. Combining this with the parameters above, $\bar{t}_a P$ can be taken as distributed,

$$N(\mu_{\bar{t}_{a_j}} \mu_{P_j}, \mu_{P_j} 2\sigma_{\bar{t}_{a_j}}{}^2 + \mu_{\bar{t}_{a_j}}{}^2 \sigma_{P_j}{}^2) . \quad (7.12)$$

The Distribution of $\sum_{j=1}^{h} \bar{t}_{a_j} P_j$ and $\sum_{j=1}^{n} \bar{t}_{a_j} P_j$

It was shown above that $\bar{t}_{a_j} P_j$ is distributed approximately as a univariate normal distribution. The problem of defining the distribution of $\sum_{j=1}^{n} \bar{t}_{a_j} P_j$, is then the problem of defining the distribution of h independent normal variates. Now if h random variables are each distributed normally, the distribution of their sum will also be normal (28, p. 72). Furthermore,

$$E\left(\sum_{j=1}^{h} \bar{t}_{a_j} P_j\right) = \sum_{j=1}^{h} E(\bar{t}_{a_j} P_j) \tag{7.13}$$

and

$$\mathrm{Var}\left(\sum_{j=1}^{h} \bar{t}_{a_j} P_j\right) \doteq \sum_{j=1}^{h} \mathrm{Var}(\bar{t}_{a_j} P_j) \ , \tag{7.14}$$

$\sum_{j=1}^{h} \bar{t}_{a_j} P_j = \bar{t}_{a_z} P_z$, can therefore be considered normally distributed with mean and variance above.

By the same method $\sum_{j=1}^{n} \bar{t}_{a_j} P_j$ can be shown to be approximately normally distributed with,

$$E\left(\sum_{j=1}^{n} \bar{t}_{a_j} P_j\right) = \sum_{j=1}^{n} E(\bar{t}_{a_j} P_j) \tag{7.15}$$

and

$$\mathrm{Var}\left(\sum_{j=1}^{n} \bar{t}_{a_j} P_j\right) \doteq \sum_{j=1}^{n} [\mathrm{Var}(\bar{t}_{a_j} P_j)]. \tag{7.16}$$

Since n in this case is the total operations performed on a machine model and

$$N = \frac{1}{H_a}(U)\ f\left(\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)\right) \tag{7.17}$$

for U assumed constant, or,

$$N = \frac{1}{H_a} f(U)\ f\left(\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)\right) \tag{7.18}$$

for U, the random variable. We next proceed to develop an expression for the distribution of U.

CHAPTER VIII

THE JOINT DISTRIBUTION WITH U

In Chapter VI, H_u was defined as the hours of equipment use. But in Chapter VII, $\bar{t}_{a_j} P_j$ was defined as the average time to complete P_j operations. Therefore, $\bar{t}_{a_j} P_j = H_{u_j}$, and $\sum_{j=1}^{h} \bar{t}_{a_j} P_j = \sum_{j=1}^{h} H_{u_j}$. In Chapter VII, it was assumed that U would be equal for all the operations within any group performed on a machine. Therefore

$$U_z \doteq \frac{h_z \bar{H}_{m_j} + \bar{H}_{u_j} h_z}{h_z \bar{H}_{u_j}} \doteq \frac{H_{m_1} + H_{u_1}}{H_{u_1}} \doteq \frac{H_{m_2} + H_{u_2}}{H_{u_2}} \doteq \qquad (8.1)$$

$$\ldots \doteq \frac{H_{m_h} + H_{u_h}}{H_{u_h}} \doteq \frac{\sum_{j=1}^{h} H_{m_j} + \sum_{j=1}^{h} H_{u_j}}{\sum_{j=1}^{h} H_{u_j}}$$

where U_z = U for the z-th group and h_z is the number of operations contained in the z-th group. Therefore,

$$E(U_1) \doteq E(U_2) \doteq \ldots \doteq E(U_h) \doteq E(U_j) \doteq E(U_z) \ , \qquad (8.2)$$

where $E(U_z)$ is the expected value of

$$U_z = \left(\sum_{j=1}^{h} H_{m_j} + \sum_{j=1}^{h} H_{u_j} \right) \div \sum_{j=1}^{h} H_{u_j} , \qquad (8.3)$$

and

$$\sigma_{U_z}^{2} \doteq \sum_{j=1}^{h} \sigma_{U_j}^{2} \left(\frac{E(\bar{t}_{a_j} P_j)}{\sum_{j=1}^{h} E(\bar{t}_{a_j} P_j)} \right)^2 \tag{8.4}$$

since operations are assumed independent and the constants of multiplication between variances of independent random variables, U_j's, are $(E(\bar{t}_{a_j} P_j)/\sum_{j=1}^{h_z} E(\bar{t}_{a_j} P_j))$, the relative frequency of individual operation occurrence.

It was shown in Equation 7.6 that

$$E(U) = \sum_{z=1}^{k} \left(\frac{E(\bar{t}_{a_z} P_z)}{\sum_{z=1}^{k} E(\bar{t}_{a_z} P_z)} \right) E(U_z)$$

and

$$\mathrm{Var}(U) \doteq \sum_{z=1}^{k} \left(\frac{E(\bar{t}_{a_z} P_z)}{\sum_{z=1}^{k} E(\bar{t}_{a_z} P_z)} \right)^2 \mathrm{Var}(U_z) \ .$$

Now substituting the expression above for $\mathrm{Var}(U_z)$,

$$\mathrm{Var}(U) = \sum_{z=1}^{k} \left(\frac{E(\bar{t}_{a_z} P_z)}{\sum_{z=1}^{k} t_{a_z} P_z} \right)^2 \sum_{j=1}^{h_z} \left(\frac{\bar{t}_{a_j} P_j}{\sum_{j=1}^{h_z} t_{a_j} P_j} \right)^2 \mathrm{Var}(U_j) \ . \tag{8.5}$$

But $\bar{t}_{a_j} P_j = H_{u_j}$, $\bar{t}_{a_z} P_z = H_{u_z}$, and $\sum_{z=1}^{k} \bar{t}_{a_z} P_z = H_u$, where H_u is the total machine hours of equipment use. Also,

$$\sum_{j=1}^{h_z} H_{u_j} = H_{u_z} \quad \text{and} \quad \sum_{z=1}^{k} H_{u_z} = H_u .$$

Therefore,

$$\sum_{z=1}^{k} \sum_{j=1}^{h_z} H_{u_j} = \sum_{j=1}^{n} H_{u_j} ,$$

where $n = \sum_{z=1}^{k} h_z$ is the total operations in all operation groups performed in the machine, and it follows that $H_u = \sum_{j=1}^{n} H_{u_j}$. In the same manner, $\sum_{j=1}^{n} H_{m_j} = H_m$. Substituting,

$$U = \frac{H_m + \sum_{z=1}^{k} \bar{t}_{a_z} P_z}{\sum_{z=1}^{k} \bar{t}_{a_z} P_z} . \tag{8.6}$$

When discussing U in Chapter VI, it was pointed out that set-up times had been disregarded in H_m. We now consider the effect of set-up time on H_m.

Set-up time occurs whenever it is necessary to change a machine over from production of one operation to production of another. If a single machine produces an operation for all items in a production lot, then the number of set-ups per year = P/Q, where Q is the size of the production lot. Standard times can be established for set-up and therefore a $\bar{t}_a$ can be established for the set-up operation. As a result in this simplest case,

$$T_{s_j} = [\frac{P_j}{Q_j}]\ \bar{t}_{s_j} = \frac{1}{Q_j} P_j \bar{t}_{s_j} \tag{8.7}$$

where T_{s_j} = total annual set-up time and Q_j is the lot size at operation j. $\bar{t}_{s_j}$ is average set-up time for operation j. Now since $1/Q_j$ is assumed constant, T_{s_j} is distributed approximately normally since $P\bar{t}_s$ has an approximately normal distribution.

In practice, however, more than one machine is often used for a single operation on production lots. Factors of immediate sales demand, scheduling relationships, and management decisions will often create variations in the number of machines assigned to successive production lots processed. However, if an expected value for the number of machines, $E(N_j)$, assigned to a specific operation could be determined, then

$$T_{s_j} = \frac{E(N_j)}{Q_j} P_j \bar{t}_{s_j}. \tag{8.8}$$

Let us assume for the present that $E(N_j)$ is known and constant and therefore T_{s_j}, a random variable, is distributed normally with mean and variance as follows:

$$E(T_{s_j}) = \frac{E(N_j)}{Q_j} E(\bar{t}_{s_j} P_j) \tag{8.9}$$

$$\sigma_{T_{s_j}}{}^2 \doteq \left(\frac{E(N_j)}{Q_j}\right)^2 \sigma_{P_j \bar{t}_{s_j}}{}^2 \tag{8.10}$$

We can now write U_T for U including set-up time and,

$$U_T = \frac{H_m + \sum_{j=1}^{n} T_{s_j} + \sum_{z=1}^{k} \bar{t}_{a_z} P_z}{\sum_{z=1}^{k} \bar{t}_{a_z} P_z} . \tag{8.11}$$

But as shown previously

$$\bar{t}_{a_z} P_z = \sum_{j=1}^{h_z} t_{a_j} P_j$$

and therefore

$$\sum_{z=1}^{k} \bar{t}_{a_z} P_z = \sum_{j=1}^{n} \bar{t}_{a_j} P_j . \tag{8.12}$$

Also, $H_m = \sum_{j=1}^{n} H_{m_j}$ and

$$U_T = \frac{\sum_{j=1}^{n} H_{m_j} + \sum_{j=1}^{n} T_{s_j} + \sum_{j=1}^{n} \bar{t}_{a_j} P_j}{\sum_{j=1}^{n} \bar{t}_{a_j} P_j} .$$

Each term in the expression for U_T has been shown to be a random variable, normally distributed for each operation j. Therefore, since each summation is a summation of normally distributed random variables, each summation will also be normally distributed (28, p. 73). Furthermore, it was shown in Chapter VII that

$$E\left(\sum_{j=1}^{n} \bar{t}_{a_j} P_j\right) = \sum_{j=1}^{n} \{E(\bar{t}_{a_j} P_j)\} \tag{8.14}$$

and

$$Var\left(\sum_{j=1}^{n} \bar{t}_{a_j} P_j\right) = \sum_{j=1}^{n} \{Var(\bar{t}_{a_j} P_j)\} \ . \tag{8.15}$$

Now

$$\sum_{j=1}^{n} T_{s_j} = \sum_{j=1}^{n} \{(\frac{E(N_j)}{Q_j})\ (P_j \bar{t}_{s_j})\}, \tag{8.16}$$

but $E(N_j)/Q_j$ has been assumed constant for operation j. Therefore, by linear combination for independent j's,

$$E\left(\sum_{j=1}^{n} T_{s_j}\right) = \sum_{j=1}^{n} \{(\frac{E(N_j)}{Q_j})\ E(P_j \bar{t}_{s_j})\}, \tag{8.17}$$

and

$$\mathrm{Var}\left(\sum_{j=1}^{n} T_{s_j}\right) \doteq \sum_{j=1}^{n} \left\{\left(\frac{E(N_j)}{Q_j}\right)^2 \mathrm{Var}(P_j \bar{t}_{s_j})\right\}. \tag{8.18}$$

It was shown on page 91 that $H_u = \sum_{j=1}^{n} (\bar{t}_{a_j} P_j)$ and in Chapter VI that H_u and H_m are each normally distributed. It was shown on page 78 that the regression of H_m on H_u is linear and $H_m = c + mH_u$, where c, the intercept, is the periodically scheduled preventive maintenance time scheduled on the basis of fixed time intervals and m is the slope of the regression line of maintenance time dependent upon running or use time. Also, $H_m = \sum_{j=1}^{n} H_{m_j}$ and $H_u = \sum_{j=1}^{n} H_{u_j}$ by definition. Also by our earlier definition of preventive maintenance scheduled on the basis of fixed time intervals the number of occurrences of such maintenance operations in a unit time period, say annually, would be constant. Therefore any variance in c must occur due to variation in time required to perform the scheduled maintenance operation. If we designate the average time per maintenance operation $\bar{t}_m$, then $\sigma_{\bar{t}_m}$ can be estimated in the same manner as $\sigma_{\bar{t}_a}$ shown earlier and

$$\sigma_c^2 \doteq \sum_{i=1}^{v} \left(\frac{H_a}{\Delta H_{t_{m_i}}}\right)^2 \sigma_{\bar{t}_{m_i}}^2 , \tag{8.19}$$

where v is the total number of time interval scheduled preventive maintenance operations on the machine per unit time period, H_a is the total hours per unit time period, and $\Delta H_{t_{m_i}}$ is the hours between individual occurrences of operation i, by schedule.

$$E(c) = \sum_{i-1}^{v} \{(\frac{H_a}{\Delta H_{t_{m_i}}})E(\bar{t}_{m_i})\} \tag{8.20}$$

and is normally distributed since it is the product of a constant and a normally distributed random variable.

By the normal assumption in linear regression that deviations from the universe line of regression are independent and normally distributed then m will also be normally, or approximately normally distributed (28, p. 657).

Assuming again that a new process is being designed, it is necessary to estimate m and its parameters. No strictly subjective means is wholly satisfactory for this problem. However, using principles similar to estimation in PERT programs, those persons available deemed best able to provide estimates may be asked to give minimum and maximum values of m which they estimate will include actual m, 90 per cent of the time. By the assumption of a normal distribution for time estimates, these values are estimates of the values $E(m) \pm 1.645\ \sigma_m$, which we will designate U_m for $E(m) + 1.645\ \sigma_m$ and L_m for $E(m) - 1.645\ \sigma_m$. Then $\hat{\sigma}_m \doteq \frac{U_m - L_m}{3.29}$, and $E(m) = \frac{U_m + L_m}{2}$.

If similar equipment, performing similar operations under similar conditions, is available, sample data may be gathered and c and m estimated using regression analysis techniques to determine E(m), Var(m), E(c), and Var(c).

Using Gardiner's approximation for the variance of the product of two random variables, the theorem E(XY) = E(X)E(Y), and linear combination rules for expected value and variance of independent random variables, with small coefficients of variation, c, m, and H_u,

$$E(H_m) = E(c) + E(m)E(H_u) \tag{8.21}$$

$$Var(H_m) \doteq Var(c) + E(m)^2 Var(H_u) + E(H_u)^2 Var(m). \tag{8.22}$$

The assumption of independence between m and H_u is justified since m has the same parameters for all values of H_u for the machine model and therefore $f(m|H_u) = f(m)$.

Returning to the expression for U_T and substituting,

$$U_T = \frac{c + m \sum_{j=1}^{n} (\bar{t}_{a_j} P_j) + \sum_{j=1}^{n} T_{s_j} + \sum_{j=1}^{n} (\bar{t}_{a_j} P_j)}{\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)}$$

$$= \frac{(m + 1) \sum_{j=1}^{n} (\bar{t}_{a_j} P_j)}{\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)} + \frac{\sum_{j=1}^{n} T_{s_j} + c}{\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)}$$

$$= (m + 1) + \frac{\sum_{j=1}^{n} (T_{s_j}) + c}{\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)}. \tag{8.23}$$

Now $(\sum_{j=1}^{n} T_{s_j} + c)$ is normally distributed since it is the sum of two normal random variables and it has been shown previously that

$\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)$ is distributed normally.

Duncan (28, p. 74) states, "since the distribution of a sum or difference is normal if the distribution of individual variables is normal, the distribution of a product or quotient will be approximately normal if the distribution of the individual variables are both normal and if the division variable does not become zero or take on values very close to zero." Now $\bar{t}_{a_j}$ will be less than .001 hours only very infrequently and $P_j \geq 100$. Therefore, $(\bar{t}_{a_j} P_j) \geq .1 > 0$, and the approximate normality of the quotient can be expected to hold. The empirical data of Tables 2 and 3 also support this hypothesis.

Since m + 1 is a normally distributed random variable plus a constant, it will be normally distributed. Now since U_T is the sum of two normally distributed random variables, it will also be normally distributed.

We now develop the expressions for the expected value and variance of U_T. Since for a normal distribution the expected value equals the mean, these will represent the parameters of the distribution of U_T.

$$E(U_T) = E(m) + 1 + \frac{\sum_{j=1}^{n} [E(T_{s_j})] + E(c)}{\sum_{j=1}^{n} [E(\bar{t}_{a_j} P_j)]} . \qquad (8.24)$$

To develop the expression for variance we first find the expression for the variance of $(\sum_{j=1}^{n} T_{s_j} + c)$, the sum of two independent normal random variables and therefore the sum of the individual variances, which from above is

$$\mathrm{Var}\left(\sum_{j=1}^{n} T_{s_j} + c\right) \doteq \sum_{j=1}^{n} \left(\frac{E(N_j)}{Q_j}\right)^2 \mathrm{Var}(P_j \bar{t}_{s_j}) + \sum_{i=1}^{v} \left(\frac{H_a}{\Delta H_{t_{m_i}}}\right)^2 \sigma_{\bar{t}_{m_i}}^{2} \qquad (8.25)$$

Gardiner (53) reduces his expression for Cov(f, g) stated earlier for the special case $f = (ax + b)/(cy + d)$ which when $a = c = 1$ and $b = d = 0$ reduces to

$$\mathrm{Var}(f) = f_0^2 \left[\frac{\mathrm{Var}(x)}{x_0^2} + \frac{\mathrm{Var}(y)}{y_0^2} - \frac{2\ \mathrm{Cov}(x,y)}{x_0 y_0}\right],$$

which is analogous to the right-hand term in the expression for U_T if x is taken to be $\sum_{j=1}^{n} T_{s_j} + c$ and y is taken as $\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)$, and

$$x_0 = E\left(\sum_{j=1}^{n} T_{s_j} + c\right) \text{ and } y_0 = E\left(\sum_{j=1}^{n} \bar{t}_{a_j} P_j\right). \qquad (8.26)$$

Looking now at the right-hand term of the expression for Var(f), $(2\ \mathrm{Cov}(x,y))/x_0y_0$, this can be written

$$\frac{2\rho \sqrt{\mathrm{Var}\ x}\ \sqrt{\mathrm{Var}\ y}}{x_0 y_0}, \qquad (8.27)$$

where ρ is the correlation coefficient between x and y. Substituting into the general expression the analogous terms above and the maximum value for ρ = 1, we obtain,

$$\frac{2\sqrt{\mathrm{Var}(\sum_{j=1}^{n} T_{s_j} + C)}\ \sqrt{\mathrm{Var}(\sum_{j=1}^{n} \bar{t}_{a_j} P_j)}}{E(\sum_{j=1}^{n} T_{s_j} + c)\ E(\sum_{j=1}^{n} \bar{t}_{a_j} P_j)} . \tag{8.28}$$

Continuing the substitution of equivalent terms from Eq. 8.25,

$$= 2\,\frac{\sqrt{\sum_{j=1}^{n} \frac{E(N_j)^2}{Q_j} \mathrm{Var}(P_j \bar{t}_{s_j}) + \sum_{i=1}^{v} \frac{H_a}{\Delta H_{t_{m_i}}}^2 \mathrm{Var}(\bar{t}_{m_i})}\ \sqrt{\mathrm{Var}(\sum_{j=1}^{n} \bar{t}_{a_j} P_j)}}{\{\sum_{j=1}^{n} [E(T_{s_j})] + E(c)\}\ \sum_{j=1}^{n} [E(\bar{t}_{a_j} P_j)]} ,$$

and the substitution of equivalent terms from Eq. 7.12,

$$= 2\sqrt{\sum_{j=1}^{n} \{(\frac{E(N_j)}{Q_j})^2 [E(P_j)^2 \mathrm{Var}(\bar{t}_{s_j}) + E(\bar{t}_{s_j})^2 \mathrm{Var}(P_j)]\} + \sum_{i=1}^{v} \frac{H_a}{\Delta H_{t_{m_i}}}^2 \mathrm{Var}(\bar{t}_{m_i})} \tag{8.29}$$

$$\cdot \sqrt{\sum_{j=1}^{n} \{E(P_j)^2 \mathrm{Var}(\bar{t}_{a_j}) + E(\bar{t}_{a_j})^2 \mathrm{Var}(P_j)\}}$$

$$\div \left[\sum_{j=1}^{n} \{\frac{E(N_j)}{Q_j} E(P_j) E(\bar{t}_{s_j})\} + \sum_{i=1}^{v} \{\frac{H_a}{\Delta H_{t_{m_i}}} E(\bar{t}_{m_i})\}\right] \sum_{j=1}^{n} \{E(\bar{t}_{a_j}) E(P_j)\}$$

Studies by Abruzzi (55, p. 109ff.) and Gomberg (56, p. 199) indicate that the coefficient of variation for operation times is much less than 1.0. For the sample data results provided by the authors the maximum value is 0.172. Abruzzi finds further that as the production rate increases, the value of the coefficient of variation tends to decrease. Empirical data in Tables 4, 5, and 6 tend to support the above findings on the value of the coefficient of variation.

We now divide the expression under consideration into two factors:

$$2\,\frac{\sqrt{\sum_{j=1}^{n}\left\{\left(\frac{E(N_j)}{Q_j}\right)^2\left[E(P_j)^2\mathrm{Var}(\bar{t}_{s_j}) + E(\bar{t}_{s_j})^2\mathrm{Var}(P_j)\right]\right\} + \sum_{i=1}^{v}\left(\frac{H_a}{\Delta H_{t_{m_i}}}\right)^2\mathrm{Var}(t_{m_i})}}{\sum_{j=1}^{n}\left\{\frac{E(N_j)}{Q_j}E(P_j)E(t_{s_j})\right\} + \sum_{i=1}^{v}\left\{\frac{H_a}{\Delta H_{t_{m_i}}}E(\bar{t}_{m_i})\right\}} \tag{8.30}$$

and

$$\frac{\sqrt{\sum_{j=1}^{n}\left\{E(P_j)^2\mathrm{Var}(\bar{t}_{a_j}) + E(\bar{t}_{a_j})^2\mathrm{Var}(P_j)\right\}}}{\sum_{j=1}^{n}\left\{E(\bar{t}_{a_j})E(P_j)\right\}}\,. \tag{8.31}$$

We analyze the second factor first. It was shown earlier that the coefficient of variation for P_j under the conditions established will not exceed approximately .0755. In the numerator of the factor we therefore have the square root of the sum of two products, each of which contains a value considerably smaller than one of the values in the product in the denominator. It can therefore be concluded that the resulting factor is much less than 1.0 under industrial conditions. For example, if $E(P_j) = 10^6$, $E(\bar{t}_{a_j}) = 1$ hour, and coefficients of variation are .0755 and 0.25, respectively. The factor value is equal to approximately .0625.

Table 4. Average Shift Efficiency--F.O.W.

Performance Level	f	Midpoint x	x^2	fx	fx^2
75- 79	3	77	5922	231	17766
80- 84	7	82	6724	574	47068
85- 89	21	87	7569	1817	158949
90- 94	29	92	8464	2664	245456
95- 99	35	97	9409	3395	329315
100-104	21	102	10404	2142	218484
105-109	6	107	11449	642	68694
110-114	3	112	12544	336	37632
Σ	125			11801	1123364

$\bar{x} = 94.41.$

$\sigma = 8.5243.$

If distribution of average annual performance rather than average daily, 250 days per year,

$$\sigma_{year} \doteq \frac{8.524}{\sqrt{250}} = 0.539$$

Table 5. Average Shift Efficiency--F.I.M.

Performance Level	f	Midpoint x	x^2	fx	fx^2
69- 69	1	67	4489	67	4489
70- 74	1	72	5184	72	5184
75- 79	2	77	5922	154	11844
80- 84	6	82	6724	492	40344
85- 89	18	87	7569	1566	136242
90- 94	20	92	8464	1840	179280
95- 99	36	97	9409	3492	338724
100-104	28	102	10404	2856	291312
105-109	8	107	11449	856	91592
110-114	3	112	12544	336	37632
115-120	1	117	13689	117	13689
Σ	124			11848	1150252

$\bar{x} = 95.5.$

$\sigma = 12.5.$

$$\sigma_{year} = \frac{12.5}{\sqrt{250}} = 0.791.$$

Table 6. Average Shift Efficiency--S.I.E.

Performance Level	f	Midpoint x	x^2	fx	fx^2
70- 74	1	72	5184	72	5184
75- 79	3	77	5922	231	17766
80- 84	2	82	6724	164	13448
85- 89	8	87	7569	696	60552
90- 94	21	92	8464	1932	177744
96- 99	33	97	9409	3201	310497
100-104	32	102	10404	3264	332928
105-109	11	107	11449	1177	125939
110-114	8	112	12544	896	100352
115-119	5	117	13689	585	68445
Σ	124			12218	1212855

$\bar{x} = 98.5.$

$\sigma = 8.88.$

$$\sigma_{year} = \frac{8.88}{\sqrt{250}} = 0.562.$$

We now analyze the first factor. The ratio $E(N_j)/Q_j$ will normally be much smaller than 1.0, since only under very abnormal conditions would one expect the number of machines required to perform an operation to approach the production lot size for the item, particularly under conditions of economic lot size determination. Therefore the related multiplier in the numerator will be less than the multiplier in the denominator, since the square of a fraction is less than the fraction. Also, by the same argument used for the second factor,

$$\sqrt{E(P_j)^2 \mathrm{Var}(\bar{t}_{s_j}) + E(\bar{t}_{s_j})^2 \mathrm{Var}(P_j)}$$

will be much less than $E(\bar{t}_{s_j})E(P_j)$. Furthermore, each element,

$$\sqrt{\left(\frac{H_a}{\Delta H_{t_{m_i}}}\right)^2 \mathrm{Var}(\bar{t}_{m_i})} < \frac{H_a}{\Delta H_{t_{m_i}}} E(\bar{t}_{m_i}), \qquad (8.32)$$

since the coefficient of variation << 1.0, and the relationship of the sums over v time interval scheduled maintenance operations will be more pronounced since the

$$\sqrt{\sum_{i=1}^{n} (x)^2} < \sum_{i=1}^{n} \sqrt{(x)^2}$$

Now since each resulting term in the numerator is much less than the corresponding term in the denominator, the multiplier 2.0 will not be expected to increase the value of the factor to something ≥ 1.0. As a result the value of the product of the two factors can be expected to be much less than 1.0 and the covariance term in the expression for the variance of U_T can be considered insignificant. From Eq. 8-23 it can be shown that,

$$\text{Var}\,(U_T) \doteq \frac{[E(\sum_{j=1}^{n} T_{s_j} + C)]^2}{[E(\sum_{j=1}^{n} \bar{t}_{a_j} P_j)]^2} \cdot$$

$$\frac{\text{Var}\,(\sum_{j=1}^{n} T_{s_j} + c)}{[E(\sum_{j=1}^{n} T_{s_j} + c)]^2} + \frac{\text{Var}\,(\sum_{j=1}^{n} \bar{t}_{a_j} P_j)}{[E(\sum_{j=1}^{n} \bar{t}_{a_j} P_j)]^2} + \text{Var}(m).$$

It has been shown above that the coefficients of variation for $\bar{t}_{a_j}$, P_j, c and T_{s_j} are small, much less than 1.0. Furthermore, if $m \geq 1.0$, $H_m \geq H_u$, which is highly unlikely in practice. It is reasonable to expect management to replace any equipment for which $H_m \rightarrow H_u$. Therefore, $U_m < 1.0$ can be expected, resulting in $\sqrt{\text{Var}(m)} <$ 1.0/3.29. As a result the coefficient of variation for m can also be expected to be much less than 1.0.

CHAPTER IX

THE DETERMINATION OF N_T

The model for N_T, total required number of machines of a specific type or model, can now be written from Eq. 7.18 as,

$$N_T = \frac{1}{H_a} f(\sum_{j=1}^{n} (\bar{t}_{a_j} P_j), U_T)$$

but $f(\sum_{j=1}^{n} (\bar{t}_{a_j} P_j), U_T)$ is just the probability density function of the product of $\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)$ and U_T, from the initial model. Substituting equivalent terms from Eq. 8.23,

$$[\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)] \, U_T = [\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)][m + 1 + \{\frac{\sum_{j=1}^{n} T_{s_j} + c}{\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)}\}] \qquad (9.1)$$

$$= [\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)][\frac{(m + 1) \sum_{j=1}^{n} (\bar{t}_{a_j} P_j) + \sum_{j=1}^{n} T_{s_j} + c}{\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)}] \qquad (9.2)$$

$$= (m + 1) \sum_{j=1}^{n} (\bar{t}_{a_j} P_j) + \sum_{j=1}^{n} T_{s_j} + c \,. \qquad (9.3)$$

It has been shown previously that the distributions of (m + 1) and $\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)$ are independent normal distributions and the distribution of their product will therefore be approximately normal. The distribution of $\sum_{j=1}^{n} T_{s_j} + c$ has also been shown to be normal. The above expression is therefore the sum of two approximately normal distributions and will therefore also be approximately normal (28, p. 72). Since $1/H_a$ is a constant and the product of a constant and a normal distribution is normal the distribution of N_T will also be approximately normal.

We now develop the expressions for the expected value and variance of N_T. Since (m + 1) is a normally distributed random variable plus a constant, E(m + 1) = E(m) + 1 and Var(m + 1) = Var(m). Also since m and $\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)$ are independent $E[(m + 1)(\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)] = E(m + 1)E[\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)]$. Also as shown earlier since m and $\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)$ are independent, the correlation coefficient = 0, and therefore the covariance = 0. Substituting this into Gardiner's (53) expression for the variance of the product of two random variables as used earlier,

$$Var[(m + 1)(\sum_{j=1}^{n} (\bar{t}_{a_j} P_j))] \doteq [E(m + 1)]^2 Var[\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)] \qquad (9.4)$$

$$+ [E(\sum_{j=1}^{n} (\bar{t}_{a_j} P_j))]^2 Var(m) .$$

Since $\sum_{j=1}^{n} T_{s_j} + c$ is the sum of two normal random variables,

$$E(\sum_{j=1}^{n} T_{s_j} + c) = \sum_{j=1}^{n} [E(T_{s_j})] + E(c), \qquad (9.5)$$

and

$$\mathrm{Var}\left(\sum_{j=1}^{n} T_{s_j} + c\right) \doteq \sum_{j=1}^{n} \left\{\left(\frac{E(N_j)}{Q_j}\right)^2 \mathrm{Var}(P_j \bar{\tau}_{s_j})\right\} + \sum_{i=1}^{n} \left(\frac{H_a}{\Delta H_{t_{m_i}}}\right)^2 \mathrm{Var}(t_{m_i}). \quad (9.6)$$

Combining the parameters of the two summed normal distributions and the constant $(1/H_a)$ (38, pp 48, 49),

$$E(N_T) = \frac{1}{H_a} \left\{ \sum_{j=1}^{n} [E(T_{s_j})] + \sum_{i=1}^{v} \left\{\left(\frac{H_a}{\Delta H_{t_{m_i}}}\right) E(\bar{t}_{m_i})\right\} + [E(m+1) \sum_{j=1}^{n} (E(\bar{t}_{a_j} P_j))]\right. \quad (9.7)$$

since

$$E(c) = \sum_{i=1}^{v} \left\{\left(\frac{H_a}{\Delta H_{t_{m_i}}}\right) E(\bar{t}_{m_i})\right\} ,$$

from Eq. 8.20, and

$$E\left(\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)\right) = \sum_{j=1}^{n} (E(\bar{t}_{a_j} P_j).$$

Also

$$\mathrm{Var}(N_T) \doteq \{E(m+1)^2 \mathrm{Var} \sum_{j=1}^{n} (\bar{t}_{a_j} P_j) + E\left(\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)\right)^2 \mathrm{Var}(m) \quad (9.8)$$

$$+ \sum_{j=1}^{n} \left(\frac{E(N_j)}{Q_j}\right)^2 \mathrm{Var}(P_j \bar{\tau}_j) + \sum_{i=1}^{v} \left(\frac{H_a}{\Delta H_{t_{m_i}}}\right)^2 \mathrm{Var}(\bar{t}_{m_i})\}\left(\frac{1}{H_a}\right)^2 .$$

Now substituting the equivalent expression for $E\left(\sum_{j=1}^{n} T_{s_j}\right)$, which is equal to $\sum_{j=1}^{n} [E(T_{s_j})]$, from Eq. 8.17,

$$E(N_T) = \frac{1}{H_a} \{ \sum_{j=1}^{n} [\frac{E(N_j)}{Q_j} E(P_j \bar{t}_{s_j})] + \sum_{i=1}^{v} [\frac{H_a}{\Delta H_{t_{m_i}}} E(\bar{t}_{m_i})] \qquad (9.9)$$

$$+ [E(m + 1) \sum_{j=1}^{n} (E(\bar{t}_{a_j} P_j))] \}$$

Further Reduction of Expressions For $E(N_T)$ and Var (N_T)

Before proceeding further, it will be recalled from Chapter V that the minimum P_j is assumed to be 100 and this occurs only when the fraction defective of the operation(s) is zero. Otherwise, P_j is equal to at least 100 plus the defective units generated. Furthermore, the maximum deviation in P_j occurs when $E(n_g) = 100$, $p' \doteq .355$, $E(n) = 155$ and $\sigma_p = .0385$. In this case the maximum deviation of p, within 95 per cent confidence limits, is .0755. This gives a maximum coefficient of variation of P_j of $\frac{.0385(155)}{155} = 0.0385$.

Also in Chapter VIII it was pointed out that Abruzzi and Gomberg found the coefficient of variation for operation times to be much less than one and the maximum value referenced by them was 0.172. They also pointed out that as the production rate increases the value of the coefficient of variation tends to decrease.

It was also shown in Chapter VIII that m is expected to be less than one and $\sqrt{Var(m)} < \frac{1.0}{3.29}$. This results in the likelihood that the coefficient of variation of m is much less than 1.0. For analysis, the maximum coefficient of variation is estimated as 1/3.29 and the minimum as .01.

The above and additional estimates of extreme input factor values for determination of $E(N_T)$ and $Var(N_T)$ are listed in Table 7. Using these values a check is now made for the relative change in $E(N_T)$ and $Var(N_T)$ for extreme conditions of the input variables.

Relative Change in $E(N_T)$ and $Var(N_T)$ Due to Individual Input Factors

We first look at the expression for $E(N_T)$. Using the values of minimum and maximum values from Table 7, ranges of individual expressions in the total expression are:

1. $$\sum_{j=1}^{n} [\frac{E(N_j)}{Q_j} E(P_j \bar{t}_{s_j})]$$

$$= \sum^{1}[(.00002)(100 \cdot 0.25)] \quad \text{to} \quad \sum_{1}^{100} [(.01)(100,000 \cdot 40)]$$

$$= 0.0005 \quad \text{to} \quad 4,000,000$$

2. $$\sum_{i=1}^{v} [\frac{H_a}{\Delta H_{t_{m_i}}} E(\bar{t}_{m_i})]$$

$$= \sum^{1}[\frac{1920}{1000}(.25)] \quad \text{to} \quad \sum_{1}^{25}[\frac{2080}{4}(100)]$$

$$= 0.48 \quad \text{to} \quad 1,300,000$$

Table 7. Factors Affecting $E(N_T)$ and/or $Var(N_T)$ with Values at Conditions Approaching Extremes for the Factor

Factor	Minimum Factor Value	Maximum Factor Value
H_a	1920 Hours	2080 Hours (Insignificant; Use 2000)
$\frac{E(N_j)}{Q_j}$	0.00002 (for Q_j = 50,000, N_j = 1)	0.01 (for P_j = 100 = Q_j, N_j = 1)
P_j	100	100,000
$\bar{t}_{s_j}$	0.25 Hour (Tape Control)	40 Hours (Major Change-Over)
$\Delta H_{t_{m_i}}$	4 Hours	1000 Hours
$\bar{t}_{m_i}$	0.25 Hour	100 Hours (Major Over-Haul)
E(m)	0.05	1.0
Var(m)	$(1/10)^2$	$(1/3.29)^2$
$\bar{t}_{a_j}$	0.001 Hour	0.2 Hour
C_{P_j}	0.0315 (@ $\bar{p}$ = 1%)	0.0385
$C_{\bar{t}_x}$	0.05	0.172
n	1	100
v	1	25

3. $[E(m + 1) \sum_{j=1}^{n} (E(\bar{t}_{a_j} P_j))]$

$= (1.05)\Sigma^{1}(.001 \cdot 100)$ to $2 \cdot \sum_{1}^{100} (0.2 \cdot 100{,}000)$

$= .105$ to $1{,}000{,}000.$

It is evident from the above that on this basis none of the factors may be eliminated from the expression for $E(N_T)$.

The range of values for the individual expressions in the expression for $Var(N_T)$ in the same manner are as follows.

1. $[E(m + 1)]^2 Var[\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)]$, where $Var(\bar{t}_{a_j} P_j) =$

$[E(P_j)]^2 Var(\bar{t}_{a_j}) + [E(\bar{t}_{a_j})]^2 Var(P_j)$, from Eq. 7.10;

$= (1.05)^2[(100)^2(.05 \times .001)^2 + (.001)^2(.0315 \times 100)^2]$

to $(2)^2[(100{,}000)^2(.172 \times .2)^2 + (.2)^2(.0385 \times 100{,}000)^2]$

$= 1.1025[(10^4 \cdot 5 \cdot 10^{-10}) + (10^{-6} \cdot 9 \cdot 9225)]$ to

$4[(10^{10} \cdot 116.964 \cdot 10^{-10}) + (.04 \cdot 14822500)]$

$= 1.1025(14.9225 \times 10^{-6})$ to $4(493016.964)$

$= 16.452 \times 10^{-6}$ to 1972068.756

Note that the values above are for n = 1 and also that $C_{\bar{t}_{a_j}}$ and C_{P_j} are unlikely to take the assigned values at the upper range. However, if we reverse the values of $C_{\bar{t}_{a_j}}$ and C_{P_j} in the minimum and maximum expressions above the range is still 364.556×10^{-6} to 5.476×10^{-8} providing a total range of 16.452×10^{-6} to 5.7465×10^{8}.

2. $[E(\sum_{j=1}^{n} (\bar{t}_{a_j} P_p))]^2 \text{Var } m$, for n = 1

$$= [.001(100)]^2(.1)^2 \quad \text{to} \quad [.2(100{,}000)]^2[\frac{1}{3.29}]^2 \text{ for } n = 1$$

$$= .0001 \quad \text{to} \quad 36.96 \times 10^6$$

3. $\sum_{j=1}^{n} [\frac{E(N_j)}{Q_j}]^2 \text{Var}(P_j\bar{t}_{a_j})$, for n = 1, and in the same manner as Eq. 7.10,

$$\text{Var}(P_j\bar{t}_{s_j}) = [E(P_j)]^2\text{Var}(\bar{t}_{s_j}) + [E(\bar{t}_{s_j})]^2\text{Var}(P_j)$$

ranges from,

$$(2 \times 10^{-5})^2[(100)^2\cdot(.05 \times .25)^2 + (.25)^2(.0315 \times 100)^2]$$

$$\text{to } 10^{-4}[(10^5)^2(.172 \times 40)^2 + (40)^2(60385 \times 10^5)^2]$$

$$= (4 \times 10^{-10})[(10^4)(125)(10^{-4}) + (.0625)(9.9225)]$$

$$\text{to } 10^{-4}[(47.3344)(10^{10}) + (1600)(.00148)(10^{10})]$$

$= 502.4808 \times 10^{-10}$ to 49.7024×10^{6}

4. $\sum_{i=1}^{v} [\frac{H_a}{\Delta H_{t_{m_i}}}]^2 \mathrm{Var}(\bar{t}_{m_i})$, for n = 1, ranges from,

$(\frac{1920}{1000})^2 (.05(.25))^2$ to $(\frac{2080}{4})^2 (.172(100))^2$

$= 5.74 \times 10^{=4}$ to 8×10^{5}.

Observation of the range of the components of the expression for $\mathrm{Var}(N_T)$ shows that each, as well as the variables of each, is too significant to be eliminated from the expression. It is therefore concluded that the expressions given for $E(N_T)$ and $\mathrm{Var}(N_T)$ cannot be reduced further in the general case.

CHAPTER X

THE EFFECT OF THE MODEL AS COMPARED TO THE SHUBIN-MADEHEIM FORMULA

The Problem

The Shubin-Madeheim formula provides the basis for present machine determination. Comparison of results using this formula and the derived model for an illustrative problem follows.

The problem can be stated as follows: Three sequential operations are all performed on the same machine model. The company expects to work 2000 hours per year, operator's mean performance has been 70 per cent of standard, and a use factor of 0.85 is used in machine determination. The following data apply to each operation.

Operation Number	Standard Time Per Piece In Hours	Expected Per Cent Defective From Operation	Estimated Annual Requirements of Good Product From Operation
1	0.013	4	
2	0.022	2-1/2	
3	0.009	3	288,000 (2,000 Gross)

Solution by Shubin-Madeheim Formula

As noted earlier, the Shubin-Madeheim formula may be written, $N = \frac{TP}{HC}$, where T is the time in standard hours, P is the required production per year, H is the expected standard hours per year and C is the use factor. H equal to 2000 x 0.70, or 1400 standard hours and C equal to 0.85 apply to all operations. P is determined for each operation by dividing the annual requirement of good product by [1 - (per cent defective/100)]. Calculating for P, to the nearest tens,

$$\text{P for operation 1} = \frac{304{,}520}{0.96} = 317{,}210 \qquad (10.1)$$

$$\text{P for operation 2} = \frac{296{,}910}{0.975} = 304{,}520 \qquad (10.2)$$

$$\text{P for operation 3} = \frac{288{,}000}{0.97} = 296{,}910 \qquad (10.3)$$

Substituting the values above and the standard times per operation into the formula provide the following individual operation requirements.

$$\text{For operation 1; } N_1 = \frac{.013(317{,}210)}{1400(0.85)} = 3.465 \qquad (10.4)$$

$$\text{For operation 2; } N_2 = \frac{.022(304{,}520)}{1400(0.85)} = 5.630 \qquad (10.5)$$

$$\text{For operation 3; } N_3 = \frac{.009(296{,}910)}{1400(0.85)} = 2.246 \qquad (10.6)$$

Total machine requirements, N_T, is equal to the sum of requirements for individual operations or 11.341. By normal convention the next higher number of machines, or 12, would be provided.

Solution by the Proposed Model

To arrive at the required number of machines by the proposed model the expressions for the mean and variance must be determined. From the Shubin-Madeheim model, $E(P_j) = P$, $\bar{t}_{a_j}$ = T/(average operator performance), and $H_a = 2000$. Other input values for each operation must be determined.

$\bar{t}_{m_1}$ = 2.05 minutes for daily lubrication with $H_{t_{m_1}}$ = 8 hours. Electrical preventive maintenance and mechanical preventive maintenance operations occur every three months. Letting $\bar{t}_{m_2}$ = 78.5 minutes for electrical and 135.7 minutes = $\bar{t}_{m_3}$ for mechanical the corresponding $H_{t_{m_2}}$ and $H_{t_{m_4}}$ can each be taken as 500 hours or one-quarter year. Standard time for set-up is 1 hour 30 minutes and average time using the same performance as for operators equals 90/.7 = 128.7 minutes. An estimate of m is taken as 0.14 by which the ratio of mean annual maintenance time will be approximately equal to 1 - C, and thereby provide a better comparison between the two methods of machine determination under similar conditions.

Using the results of the research of Abruzzi and Gomberg referenced earlier, the coefficient of variation for the preventive maintenance and set-up mean times is taken as .03. Note that by so doing the effect of variation is small. If the coefficients of variation were larger, any discrepancies between the proposed and traditional models would also be larger.

As an estimate of N_j for each operation we use the individual operation machine requirements as determined by the traditional model. To estimate Q_j we determine the economic production lot size assuming a unit value of $0.50, storage and carrying charges equal to 25 per cent of inventory value, and order placement cost of $28. Using a simplified economic lot size quantity formula (57, p. 5.29)

Table 8. Value of Mean Time for Daily Lubrication and Estimate of Standard Deviation of Mean Time

Average Time (Minutes)	Deviation from Mean $d = (\bar{t}_{m_i} - t_{m_i})$	d^2
2.05	0	0
2.00	-.05	.0025
1.95	-.05	.0025
2.04	-.10	.0100
2.10	.05	.0025
1.90	-.15	.0225
2.00	-.05	.0025
2.10	.05	.0025
2.00	-.05	.0025
2.10	.05	.0025
2.00	-.05	.0025
2.05	0	0
2.30	.25	.0625
2.10	.05	.0025
2.10	.05	.0025
Σ 32.79		.1201

$\bar{t}_{m_i} = 2.05$ Minutes.

$\hat{\sigma}_{\bar{t}_{m_i}} \doteq 0.087.$

$C_{\bar{t}_{m_i}} = 0.042.$

$$Q = \sqrt{\frac{AD}{KB}} \tag{10.7}$$

where

A = Order cost.

D = Annual usage.

K = Annual carrying charge as fraction of value.

B = Unit value.

To find the quanity to start, add the expected loss during manufacturing, rounding off to 1000's. Values are collected in Table 9. $E(N_i)$ has been taken as the number of machines for operation, i, as determined by the traditional formula and the coefficient of variation for production operation times is taken as 0.1. Also note that by Eq. 5.14 the variance of P_j is estimated as $Q\hat{p}_p(1 - \hat{p}_p)$. Since the variance of P_j is based upon the expected input quantity to the system Q, assumed constant, and the estimated fraction defective for prior operations, $\hat{p}_p$, then $Var(P_1)$ would be zero for the model.

Table 9. Input Values for Example of Model Results

Variable	Value
H_a	2,000 Hours
$E(P_1)$	317,210
$E(P_2)$	304,520
$E(P_3)$	296,910
$\bar{t}_{a_1}$	0.0185 Hour

Table 9. Input Values for Example of Model Results (Continued)

Variable	Value
$\bar{t}_{a_2}$	0.0314 Hour
$\bar{t}_{a_3}$	0.0129 Hour
$E(t_{s_j})$	2.145 Hours
$E(t_{m_1})$	0.034 Hour
$E(t_{m_2})$	1.308 Hours
$(E(t_{m_3})$	2.262 Hours
$\Delta H_{t_{m_1}}$	8 Hours
$\Delta H_{t_{m_2}}$	500 Hours
$\Delta H_{t_{m_3}}$	500 Hours
m	0.14
$Var[\sum_{j=1}^{n} (\bar{t}_{a_j} P_j)] =$	344379 + 886485 + 146012 = 1376876
Var(m)	0.04
$Var(P_1 \bar{t}_{s_1}')$	416,576,670
$Var(P_2 \bar{t}_{s_2})$	383,968,305
$Var(P_3 \bar{t}_{s_3})$	365,052,970
$Var(\bar{t}_{m_1})$	1.96×10^{-6}
$Var(\bar{t}_{m_2})$	0.0015
$Var(\bar{t}_{m_3})$	0.0046
$Var(t_{s_j})$	0.00414
$E(N_1)$	3.465
$E(N_2)$	5.630
$E(N_3)$	2.246

Table 9. Input Values for Example of Model Results (Continued)

Variable	Value
Q_1	9,000
Q_2	8,640
Q_3	8,420
$C_{\bar{t}_{a_j}}$	0.100
$Var(\bar{t}_1)$	0.34225×10^{-5}
$Var(P_1)$	0
$Var(\bar{t}_2)$	0.95596
$Var(P_2)$	12180.86
$Var(\bar{t}_3)$	0.16641×10^{-5}
$Var(P_3)$	19278.438

Example Results

Applying the values in Table 9 and above to Eq. 9.9 for $E(N_j)$, the estimate of the mean of the distribution, we obtain the following:

$$E(N_T) = \frac{1}{H_a}\left\{\sum_{j=1}^{n}\left[\frac{E(N_j)}{Q_1} E(P_j \bar{t}_{s_j})\right] + \sum_{i=1}^{v}\left[\frac{H_a}{\Delta H_{t_{m_i}}} E(\bar{t}_{m_i})\right] \right. \tag{10.8}$$

$$\left. + E(m+1) \sum_{j=1}^{n} (E(\bar{t}_{a_j} P_j))\right\}$$

$$= \frac{1}{2000}\left\{\left[\frac{3.465}{9000}(680{,}415) + \frac{5.63}{8420}(643{,}195)\right.\right.$$

$$\left. + \frac{2.246}{8420}(529{,}622)\right] + \left[\frac{2000}{8}(0.034) + \frac{2000}{500}(1.308)\right.$$

$$\left.\left. + \frac{2000}{500}(2.262)\right] + 1.14\,[5868.39 + 9561.93 + 3830.14]\right\}$$

$$= \frac{1}{2000}\{[261.960 + 419.097 + 141.273] + [8.500$$

$$+ 5.232 + 9.058] + [21956.924]\}$$

$$= \frac{1}{2000}(22802.034)$$

$$= 11.401 \text{ machines.}$$

$$\mathrm{Var}(N_T) \doteq \frac{1}{(2000)^2}\{[E(m+1)]^2 \mathrm{Var}[\sum_{j=1}^{n}(\bar{t}_{a_j}P_j)] \tag{10.9}$$

$$+ [E(\sum_{j=1}^{n}\bar{t}_{a_j}P_j)]^2 \mathrm{Var}(m) + [\sum_{j=1}^{n}(\frac{E(N_j)}{Q_j})^2 \mathrm{Var}(P_j\bar{t}_{a_j})]$$

$$+ \sum_{i=1}^{v}(\frac{H_a}{\Delta H_{t_{m_i}}})^2 \mathrm{Var}(\bar{t}_{m_i})\}$$

$$+ \frac{1}{(2000)^2}\{[(1.4)^2(1376876)] + [(19260.46)^2(.04)]$$

$$+ [\frac{3.465}{9000}^2(416,576,670) + \frac{5.63}{8640}^2(383,968,305)$$

$$+ {}^2(365,052,970)] + [(250)^2(1.96 \times 10^{-6})$$

$$+ 4^2(.0015) + 4^2(.0046)]$$

$$= \frac{1}{(2000)^2}[1789938 + 14838612.776 + 62.743$$

$$+ 162.227 + 26.612 + .1225 + .0240 + .0736]$$

$$= \frac{1}{(2000)^2}(16628802.582) = \frac{16628802.582}{4 \times 10^6}$$

$$= 4.157$$

Standard Deviation $N_T = \sqrt{\mathrm{Var}(N_T)} = 2.04$

The Effect of the Variance on Economic Decision

Note that the results by the Shubin-Madeheim model for N and by the proposed model for $E(N_T)$ are similar. In either case it would be necessary to provide 12 machines to satisfy estimated requirements. However, by the variance factor of the model a further economic decision is possible. To accomplish this, use is made of the Morris decision model (8, pp. 35-39 and 6, pp. 225-226) referenced to earlier in Chapter II. Repeating Morris' decision model with present notation,

$$C_0(N_T) = C_{0_1}M + C_{0_2}\int_{N_T}^{\infty}(n_t - N_T)f(n_t)dn_t \qquad (10.11)$$

$C_0(N_T)$ = The expected cost of a policy of providing N_T machines.

C_{0_1} = Fixed charges per machine per period.

C_{0_2} = Cost penalty (excess over regular time) per machine of overtime production.

$f(n_t)$ = Probability distribution of actual number of machines required in a period.

The optimum policy is then computed by setting

$$\frac{dC_0(N_T)}{d(N_T)} = 0.$$

The result is a value of N_T taken as N_T^*, which satisfies the relationship,

$$\int_0^{N_T^*} f(n_t)dn_t = F(N_T^*) = \frac{C_{0_2} - C_{0_1}}{C_{0_2}} \qquad (10.12)$$

In the decision model costs are assumed to be linear.

An application of the decision model is now made to the example problem. A plant in Indianapolis, Indiana estimates the cost per machine hour on regular time as \$7 and on overtime as \$11. Using these cost values and the mean and variance of N_T obtained above, a decision as to the economic number of machines can be made. Rewriting Eq. (10.12) and then substituting values for symbols,

$$\int_0^{N_T^*} f(n_t)dn_t = F(N_T^*) = \frac{C_{0_2} - C_{0_1}}{C_{0_2}} = \frac{11 - 7}{11} = \frac{4}{11} \tag{10.13}$$

$$= P\,\{n_t \leq N_T^*\}\ .$$

The Z value of the standardized normal distribution can be determined,

$$Z = \frac{N_T^* - E(N_T)}{\sqrt{Var(N_T)}}\ . \tag{10.14}$$

At $P\,\{n_t \leq N_T^*\} = \frac{4}{11} = .363$, $Z = -.35$.

Substituting in Eq. 10.14,

$$-.35 = \frac{N_T^* - 11.401}{2.04}$$

$$N_T^* = 11.401 - .714 = 10.687.$$

The saving in this case under the stated decision rule would be one machine. It should be repeated that variances on operation, maintenance, and set-up times were assumed small. Increased variances, an increased value for m, or reduced lot sizes could significantly affect the resultant optimal value of N_T as compared to $E(N_T)$.

The Effect of Variance on Decisions of Availability

Perhaps another management decision rule as important in many cases as that of optimal economy is the ability to assure sufficient production capacity. This basis of decision is reinforced by the fact that during periods of maximum machine or plant capacity demand costs tend to rise since the minimum operating cost point may be at 85 to 95 per cent capacity. By the model, and in fact any of the available models, the resultant expected number of machines assumes a level of production capacity. Furthermore the economic decision rule of Morris assumes time availability for overtime which may not be true for a plant operating on a 24-hour day. To overcome the effect of these conditions, management may wish to provide a satistical confidence of sufficient capacity.

Statistical confidence may be obtained by again applying the Z value to the standardized normally distributed random variable N_T. For any desired confidence the corresponding Z-value can be obtained from the tables of the cumulative probabilities of the normal probability distribution such as that of Duncan (28, p. 869). For example, if a confidence level of 0.90 is desired, the corresponding Z-value of

1.282 can be used to determine the corresponding N_T. For the data used in the example:

$$\text{For } P\ \{n_t \leq N_T\} = .90,\ Z = 1.282$$

and

$$Z = \frac{N_T - E(N_T)}{\sqrt{Var(N_T)}} .$$

Substituting,

$$1.282 = \frac{N_T - 11.401}{2.04} ,$$

and

$$N_T = 1.282(2.04) + 11.401$$

$$= 2.615 + 11.401$$

$$= 14.016 \text{ machines.}$$

At this point another decision must be made as to whether to reduce confidence and provide 14 machines or increase confidence by providing 15 machines.

In the same manner as for the desired confidence of 0.90, the obtainable confidence can be determined. First for $N_T = 14$,

$$Z = \frac{N_T - E(N_T)}{\sqrt{Var(N_T)}} = \frac{14 - 11.401}{2.04} = \frac{2.599}{2.04} = 1.274$$

and the corresponding confidence level from the cumulative probabilities table is 0.8987. For $N_T = 15$,

$$Z = \frac{N_T - E(N_T)}{\sqrt{Var(N_T)}} = \frac{15 - 11.401}{2.04} = \frac{3.599}{2.04} = 1.764,$$

and the corresponding confidence level is 0.9611.

CHAPTER XI

CONCLUSIONS AND OBSERVATIONS

The objective of the proposed research was to develop a procedure for making a better estimate of machine and equipment requirements for a new manufacturing activity than was possible with previously published techniques. This study has accomplished this objective by deriving a model which recognizes and incorporates the random nature of those variables affecting machine requirements. This has been accomplished by combining variables in an approximating normal distribution. Use of the model for decisions based upon economic optimality or desired level of confidence in meeting a fixed production demand has been illustrated.

The proposed model incorporates the following features in addition to the treatment of all input variables except the forecast demand as random variables.

1. The use of average time rather than standard time values for operational time. Average times based upon historical data directly or from standard time and historical operator performance levels against standards permits the use of a normal approximating distribution, thereby simplifying the analysis problem.

2. The use of one year as the basic time period rather than the one day normally found in earlier models. This prevents an arbitrary reduction of annual cycle factors to daily values.

3. The definition of the use factor as the limiting value of

the utilization factor. This permits the use in the model of a factor which is independent of demand for equipment time which may have a major effect on equipment utilization.

4. The use of the reciprocal rather than the use factor, thereby securing a product of major factors rather than a ratio. The use of a product provides a vehicle for deriving the approximating joint distribution of the number of machines.

5. The breakdown of (1 - use factor) into its major components, maintenance time and set-up time. Maintenance time is further divided into those components dependent upon real time and running time. The separation of real time dependent maintenance permits the estimation of minimum maintenance time per machine which then serves as an estimate of the intercept of a least squares regression equation of total maintenance time on use time.

Although further empirical studies are desirable to reinforce the nature of both the resultant and input variable distributions, logical and mathematical derivation support the appropriateness of the model. The availability of the model will permit facility planners to better analyze the effect of the randomness of input variables on equipment provisioning. Probabilistic evaluation of cost and capacity effects of alternative numbers of machines will be possible during plant planning and design.

Limitations in Application of the Model

The steps involved in establishing and operating a new manufacturing facility may be reduced to the following:

1. Determination of product mix to be produced followed by the forecast of demand for each product.

2. Design of the production process for each product.

3. Determination of requirements and installation of machines and equipment.

4. Operation of facilities under dynamic industrial and general economy conditions.

This research was concerned only with the third step. It is therefore assumed that the decisions relative to demand and production methods are fixed and known. If a particular sales forecasting model is used by management it may be combined with the derived model to determine machine requirements. This will result in a model specifically adapted to the needs of the individual firm. When it is desired to delay the decision as to the production method to be used until after the determination of machine requirements by the model it will be necessary to establish total machine requirements for each alternative production method and then compare costs for the total production system. The derived model is not designed to select the optimal production method directly.

The model does not provide for future dynamic changes in demand. The model may be used for determination of machine requirements at fixed levels of demand but determination of the schedule for facility changes must be accomplished by other means.

This model is not recommended for use for continuous production processes. In this case the sequence of machine or equipment arrangement tends to be fixed by the order of the production operations and techniques of line balancing are more efficient for the problem.

Suggestions for Further Research

Opportunities for further research exist primarily in empirical analysis of input factors to the proposed model. A partial listing would include the following.

1. The regression curve of annual maintenance time on annual running time with analysis of variance for a number of machine types. This would involve a long period of time to gather data unless satisfactory data are available in historical records.

2. Incorporation of forecast demand as a random variable in the model. This will probably result in a unique joint distribution for each probabilistic forecasting model.

3. Incorporation of the effect of master schedules on the model.

4. Use of the model under other decision rules such as the minimax and minimum regret decision principles.

5. The effect of a variety of lot size policies including investment or space constraints on the model.

6. For an existing plant the effect of space constraints in the manufacturing area on the model.

7. Empirical analysis of the proposed model.

8. Investigation of the relative goodness of fit of stochastic distributions for maintenance times rather than the normal approximation used in the model.

9. The derivation of decision procedures which will incorporate

dynamic changes over future planning periods.

10. The incorporation into decision methods of cost factors other than costs of regular and overtime production. A specific suggestion is the effect of seasonal variations in demand. The proposed model inherently assumes constant demand on a machine model.

APPENDIX

APPENDIX I

GLOSSARY OF TERMS

A = Performance against standard.

C = Factor of use of equipment.

c = The intercept in the model $H_m = c + mH_u$, equivalent to time interval scheduled maintenance time per unit time period per machine.

c_i = Number of defectives generated by operation i.

C_{0_1} = Fixed charges per machine per period.

C_{0_2} = Cost penalty (excess over regular time) per machine period of overtime production.

C_x = Coefficient of variation for random variable x.

d = Number of defective items.

$E(x)$ = Expected value of the variable x.

H = Standard hours per time period.

H_a = Actual hours per time period per machine.

H_m = Hours per time period lost due to maintenance.

H_u = Hours of equipment use per time period.

L_m = $E(m) - 1.645_m$.

m = The slope in the model $H_m = c + mH_u$.

N = Number of machine per operation.

N_I = An incomplete determination of N.

N_j = The number of machines assigned to operation j.

N_T = Total requirement of a specific machine model or type.

n = Number of items passing through an operation.

n_g = Number of good items resulting from an operation.

P = Operation cycles per time period.

p = Fraction defective.

p' = Fraction defective of the universe.

$\bar{p}$ = Average fraction defective.

p'_p = Fraction defective generated by a process prior to the operation of interest.

Q = Quantity of items started through a production process.

Q = Size of an individual production lot.

r = The correlation coefficient between two random variables.

sa = Forecast annual product sales.

s_x^2 = A sample estimate of the variance of the random variable x.

$s_{\bar{t}_a}^2$ = An estimate of the variance of the distribution of $\bar{t}_a$.

T = Standard time per operation cycle.

T_s = Total annual set-up time for a machine model per machine.

t_a = Actual time per operation cycle.

$\bar{t}_a$ = Sample average of t_a.

t_m = Operation time for a maintenance operation scheduled on the basis of a fixed time interval.

U = The use factor reciprocal = 1/C.

U_m = $E(m) + 1.645_m$.

U_T = Reciprocal of the use factor including set-up time.

u_j = Number of part j in the final product.

v = The total number of time interval schedule preventive maintenance operati-ns on the machine per unit time period.

$\bar{x}$ = The sample average value of the variable x.

$\hat{x}$ = An estimate of x.

z = Operation group.

$\Delta H_{t_{m_i}}$ = Total hours between occurrences of maintenance operation i, scheduled on the basis of a fixed time interval.

μ_x = The mean of the random variable x.

$\sigma^2_{t_a}$ = The variance of the population of t_a.

σ_x = Standard deviation of the variable x.

BIBLIOGRAPHY

BIBLIOGRAPHY

1. Muther, Richard, *Practical Plant Layout.* New York; McGraw-Hill Book Co., 1955.

2. Council of Economic Advisors, *Economic Indicators.* Washington: U. S. Government Printing Office (pamphlet).

3. Grant, Eugene L., and W. Grant Ireson. *Principles of Engineering Economy.* Fourth edition. New York: The Ronald Press Co., 1960.

4. Thuesen, H. G., *Engineering Economy.* Second edition. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1957.

5. Terborgh, George, *Dynamic Equipment Policy.* New York: McGraw-Hill Book Co., 1949.

6. Morris, William T., *Engineering Economy.* Homewood, Illinois: Richard D. Irwin, Inc., 1960.

7. Johnston, A. J. C., "The Calculation of Equipment Capacities," *Time and Motion Study.* Vol. 10, No. 2 (February, 1961), pp. 29-31.

8. Morris, William T., "Facilities Planning," *Journal of Industrial Engineering.* Vo. IX, No. 5 (September-October, 1958), pp. 358-361.

9. Shubin, John A., and Huxley Madeheim, *Plant Layout.* New York: Prentice-Hall, Inc., 1951.

10. Ireson, William Grant, *Factory Planning and Plant Layout.* New York: Prentice-Hall, Inc., 1952.

11. Apple, James M., *Plant Layout and Materials Handling.* New York: The Ronald Press Co., 1950.

12. Reed, Ruddell, Jr., *Plant Layout: Factors, Principles, and Techniques.* Homewood, Illinois: Richard D. Irwin, Inc., 1961.

13. Fetter, Robert B., "Some Application of Probability Theory to Manufacturing Problems," *Industrial Management Reprint Number 26.* Cambridge: Massachusetts Institute of Technology.

14. Moore, James M., *Plant Layout and Design.* New York: The Macmillan Co., 1962.

15. Schiller, D. H., and M. M. Lavin, "The Determination of Requirements

for Warehouse Dock Facilities," *Operations Research.* Vol. 4, (April, 1956) pp. 231-242.

16. Morris, William T., *Analysis for Materials Handling Management.* Homewood, Illinois: Richard D. Irwin, Inc., 1962.

17. Metzger, Robert W., *Elementary Mathematical Programming.* New York: John Wiley & Sons, Inc., 1958.

18. Bowman, Edward, H., and Robert B. Fetter, *Analysis for Production Management.* Revised edition. Homewood, Illinois: Richard D. Irwin, Inc., 1961.

19. Bierman, Harold, Jr., L. E. Fouraker, and R. K. Jaedicke, *Quantitative Analysis for Business Decisions.* Homewood, Illinois: Richard D. Irwin, Inc., 1961.

20. Reinfeld, Nyles V., and William R. Vogel, *Mathematical Programming.* Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1958.

21. Flagle, Charles D., William H. Huggins, and Robert H. Roy, (eds.) *Operations Research and Systems Engineering.* Baltimore: The Johns Hopkins Press, 1960.

22. Salveson, M. E., "The Assembly Line Balancing Problem," *Journal of Industrial Engineering.* Vol. VI, No. 3 (May-June, 1955), pp. 18-25.

23. Vazsonyi, Andrew, *Scientific Programming in Business and Industry.* New York: John Wiley & Sons, Inc., 1958.

24. Helgeson, W. B., and D. P. Birnie, "Assembly Line Balancing Using the Ranked Positional Weight Technique," *Journal of Industrial Engineering.* Vol. XII, No. 6 (November-December, 1961), pp. 394-398.

25. Eilon, Samuel, *Production Planning and Control.* New York: The Macmillan Co., 1962.

26. Nadler, Gerald, *Work Design.* Homewood, Illinois: Richard D. Irwin, Inc., 1963.

27. Barnes, Ralph M., *Work Sampling, 2nd ed.* New York: John Wiley & Sons, Inc., 1957.

28. Duncan, Acheson J., *Quality Control and Industrial Statistics,* Revised edition. Homewood, Illinois: Richard D. Irwin, Inc., 1959.

29. Walls, R. T., "How to Measure and Control Job Shop Labor Performance," *American Machinist,* Oct. 5, 1959.

30. Forrester, George, *A Statistical Analysis of Some of the Causes of Timed Variance in Stop Watch Studies,* Unpublished Master's Thesis,

Georgia Institute of Technology, 1953.

31. Peñuela, L. J., Jr., *An Experimental Evaluation of the Presence of an Analyst on Work Performance Times in Time Study Analysis*, Unpublished Master's Thesis, Georgia Institute of Technology, 1961.

32. Abruzzi, Adam, "Developing Standard Data for Predictive Purposes," *Journal of Industrial Engineering*, Vol. II, No. 3, p. 15 (Nov. 1952).

33. Kennedy, M. H., *An Investigation of the Pattern of Variation of Cycle Performance Times for a Repetitive Manual Operation*, Unpublished Master's Thesis, Georgia Institute of Technology, 1957.

34. Ostle, B., *Statistics in Research, 2nd ed.* Ames, The Iowa State University Press, 1963.

35. Brown, R. G., *Statistical Forecasting for Inventory Control.* New York· McGraw-Hill Book Co., 1959.

36. Winters, P. R., "Forecasting Sales by Exponentially Weighted Moving Averages," *Management Science* 6:324-342 (1960).

37. Sheele, E. D., W. L. Westerman, and R. J. Wimmert, *Principles and Design of Production Control Systems.* Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1960.

38. Bowker, A. H. and G. J. Lieberman, *Engineering Statistics.* Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1959.

39. Koff, R. M., "What the Experts Tell Us about Mechanical Reliability," *Product Engineering*, Nov. 16, 1959, pp. 21-24.

40. Lloyd, D. K. and M. Lipow, *Reliability: Management, Methods, and Mathematics.* Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1962.

41. ________, "How You Can Analyze Standby Equipment Needs," *Plant Engineering*, April, 1959, pp. 111-113.

42. Flagle, C. D., W. H. Huggins, R. H. Roy, *Operations Research and Systems Engineering.* Baltimore: The Johns Hopkins Press, 1960.

43. Feller, W., *An Introduction to Probability Theory and Its Application, 2nd ed.* New York: John Wiley & Sons, Inc., 1957.

44. Wadsworth, G. P. and J. G. Bryan, *Introduction to Probability and Random Variables.* New York: McGraw-Hill Book Co., Inc., 1960.

45. Lindgren, B. W. and G. W. McElrath, *Introduction to Probability*

and Statistics. New York: The MacMillan Co., 1959.

46. Cramer, Harold, *Mathematical Methods of Statistics*. Princeton: Princeton University Press, 1946.

47. Kendall, M. G. and A. Stuart, *The Advanced Theory of Statistics*, Vol. 1. New York: Hafner Publishing Co., 1958.

48. Mood, A. M., *Introduction to the Theory of Statistics*. New York: McGraw-Hill Book Co., Inc., 1950.

49. Hald, A., *Statistical Theory with Engineering Applications*. New York: John Wiley & Sons, Inc., 1952.

50. Arkin, H. and R. R. Colton, *Statistical Methods*. New York: Barnes and Noble, Inc., 1939.

51. Mood, A. M. and F. A. Graybill, *Introduction to the Theory of Statistics, 2nd ed.* New York: McGraw-Hill Book Co., Inc., 1963.

52. Burington, R. S. and D. C. May, Jr., *Handbook of Probability and Statistics with Tables*. Sandusky, Ohio: Handbook Publishers, Inc., 1953.

53. Gardiner, Donald A., "Approximate Variability of Nonlinear Forms," Unpublished Paper.

54. Fraser, D. A. S., *Statistics—An Introduction*. New York: John Wiley & Sons, Inc., 1958.

55. Abruzzi, Adam, *Work Measurement—New Principles and Procedures*. New York: Columbia University Press, 1952.

56. Gomberg, William, *A Trade Union Analysis of Time Study*. Chicago: Science Research Associates, 1948.

57. Carson, G. B. (ed.), *Production Handbook, Second Ed.* New York: Ronald Press Co., 1958.

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